Magnetic Resonance Imaging

Introduction to the physical principles of MRI

Organization

- **Lectures**
  - We,12:30-14:00
  - Room: 4263|201 (26 C 201)
- **Lecturer**
  - Prof. N. Jon Shah
  - Dr. Tony Stöcker
- **Practial Training:**
  - Two consecutive days at the end of Semester
  - Date to be announced / discussed
- **Oral Examination:**
  - Two days in the semester break
  - Dates to be announced / discussed

Organization

- **Meet the lecturer**

Prof. N. Jon Shah
n.j.shah@fz-juelich.de

Dr. Tony Stöcker
t.stoecker@fz-juelich.de

Office Hours: ==> after lecture

Organization

- For your self studies:
  - E.M. Haacke et al, Magnetic Resonance Imaging: Physical Principles and Sequence Design, John Wiley 1999
  - Z.P. Liang, P.C. Lauterbur, Principles of Magnetic Resonance Imaging: A Signal Processing Perspective, SPIE 1999
  - M.T. Vlaardingerbroek, J.A. den Boer, Magnetic Resonance Imaging: Theory and Practice, Springer, 2003
  - J.P. Hornak, Ph.D., The Basics of MRI,
    http://www.cis.rit.edu/htbooks/mri/

- Lecture materials:
  - Slides, and other information in the virtual learning room (L2P):
    http://www.elearning.rwth-aachen.de
  - Login using your student account
  - Registration in CAMPUS office for this course is necessary:
    - *e.g.: CAMPUS > Dozentensuche „Shah“*
    - *=> Lehrveranstaltungen*
    - *=> Experimentelle Magnetresonanztomographie*

Course Overview

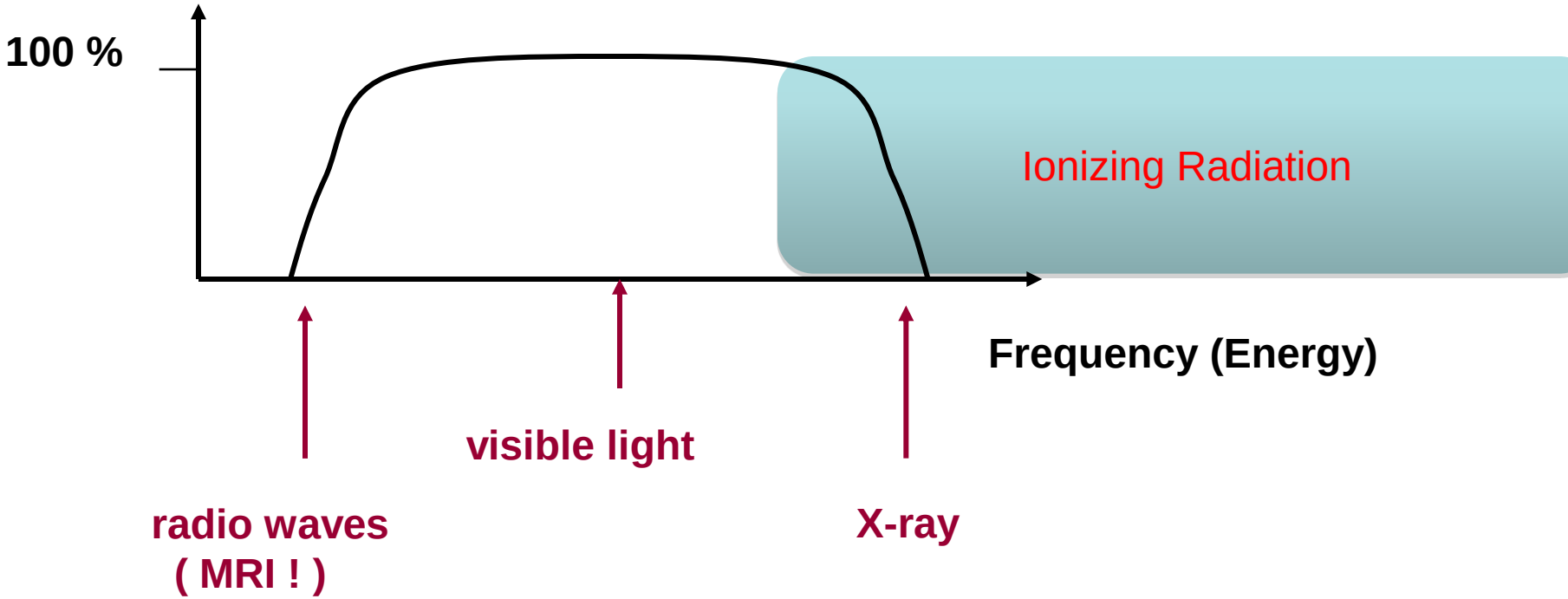
1. Introduction and Overview
2. Math Basics
3. MR Fundamentals
4. Excitation
5. Imaging Principles
6. Sequences and Contrasts
7. Rapid Imaging
8. Parallel Imaging
9. Hardware
10. Selected Applications

Outline

- Introduction
  - Magnetic Resonance Imaging
  - Appetiser – some Examples
- MR Scanner Components
- Physical Introduction
  - Spin ½ in B_0
  - Larmor Precession
  - Resonance (excitation and signal reception)
  - Relaxation
  - FID, Echoes and Fourier Transform
  - Basic Idea of MR Imaging
- Basic Safety Issues
- Sum-up

MRI ≠ X-ray

EM absorbtion in the human body:



Magnetic Resonance Imaging

MRI is a tomography method but it is very different from CT (Computer Tomography)!
Some positive characteristics of MRI:

Non-ionising	*Using radio-waves, not X-rays*
Non-invasive	*No contrast agents required*
Soft-tissue sensitive	*MRI signal ~ Proton density ρ ~ H_2O*
Quantitative	*MRI signal ~ Magnetisation = f(ρ,T_1, T_2, T_2*, D, α, …)* *➔ density, relaxometry, diffusivity, perfusion, …*
Multi-contrast	*Various imaging sequences/ parameters*
Multi-purpose/modal	*Anatomy, activity, connectivity, vessels, …*
Oblique slices	*Flexible spatial encoding with arbitrary image slice orientation.*

MRI's weakness:
low signal-to-noise ratio *and* usual MRI not usable for material with little water content!

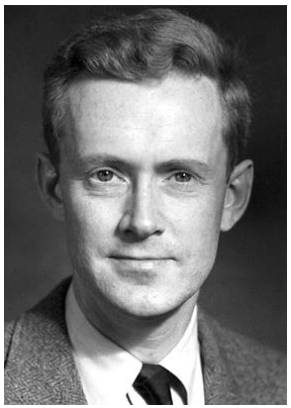
Discovery of NMR and MRI

Nuclear spin manipulation through external fields:

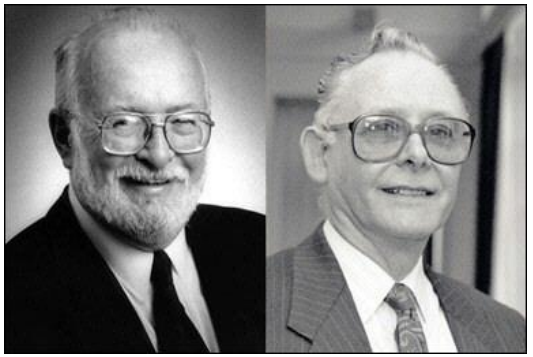
- Static magnetic field: $B_0 \cdot e_z \Rightarrow$ spin polarization
- Radiofreq. field: $B_1(t) \cdot \boldsymbol{e}_{xy} \Rightarrow$ resonance signal
- Gradient field: $(\boldsymbol{G}(t) \cdot \boldsymbol{r}) \cdot \boldsymbol{e}_z \Rightarrow$ spatial frequencies

➔ **Imaging!**

Felix Bloch
(1905-1983)

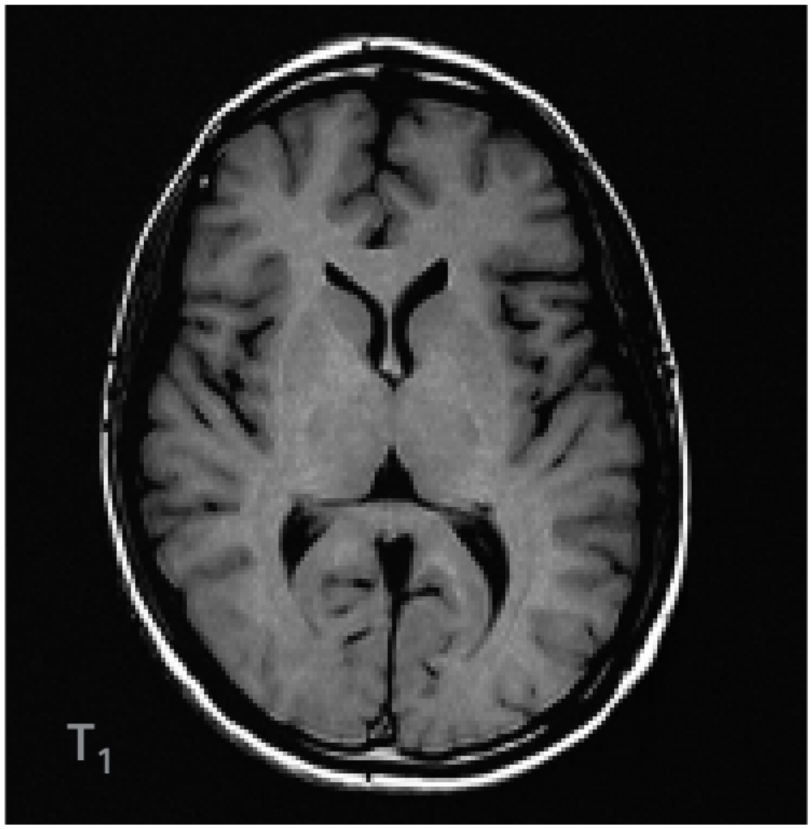


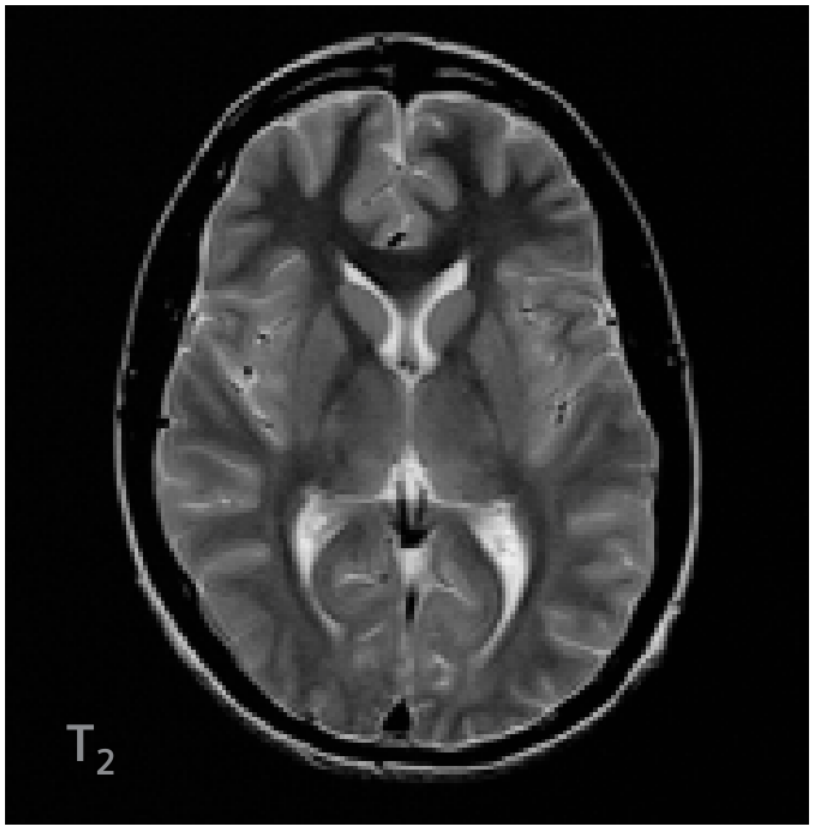
Edward Purcell



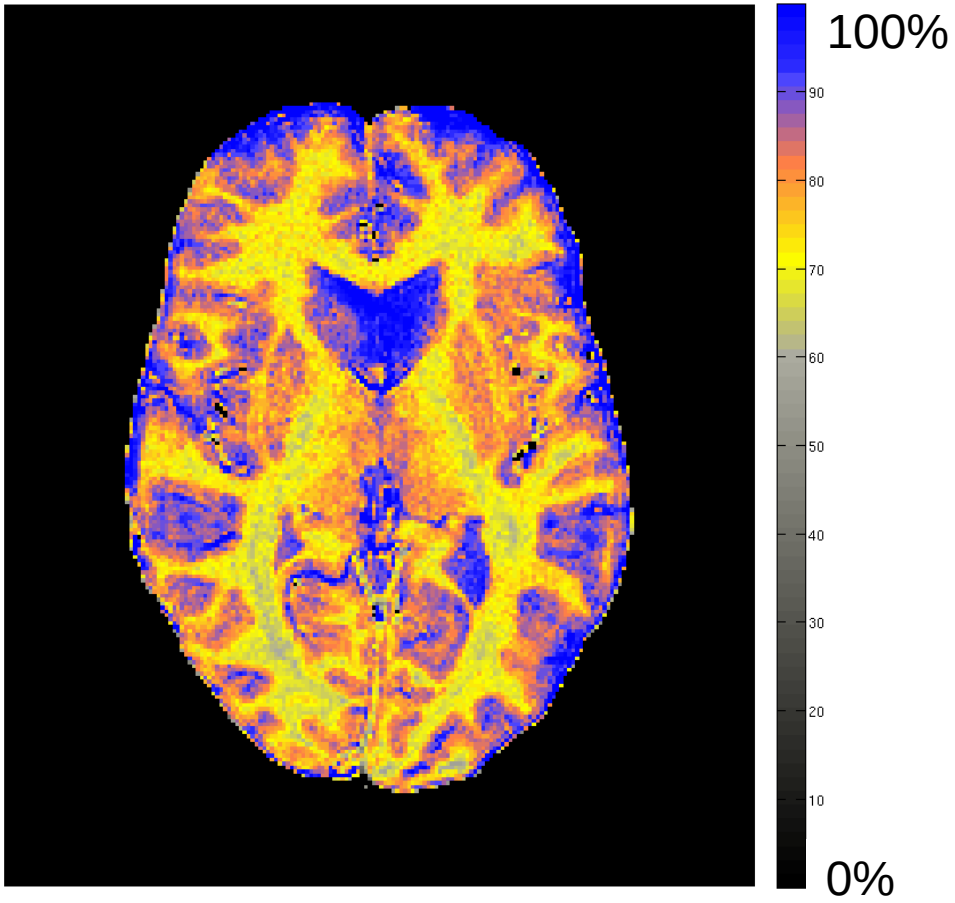
Paul Lauterbur and Sir Peter Mansfield

Examples: Multi-Contrast (T1/T2-weighting)





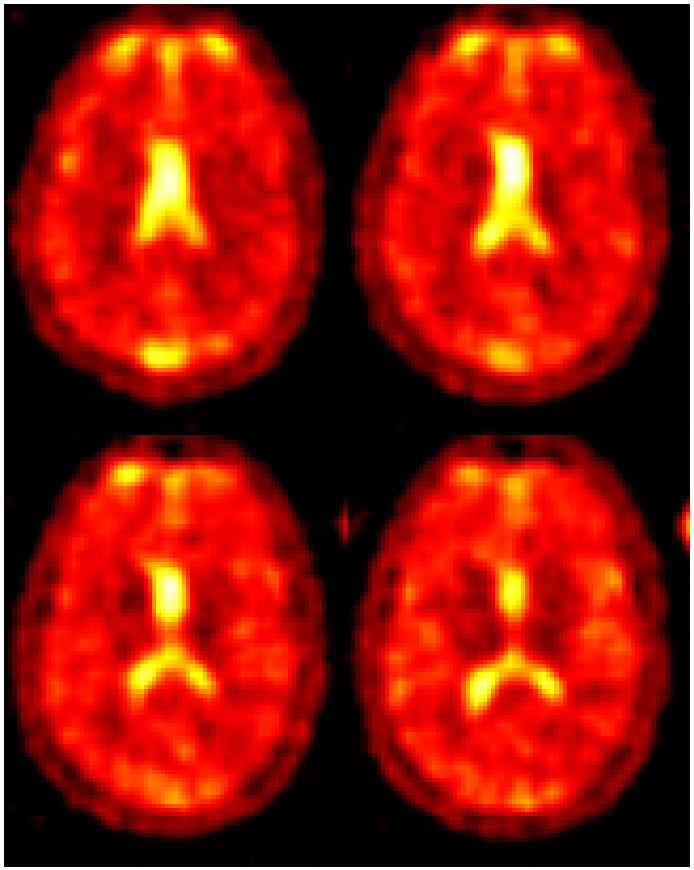
Examples: Quantitative – Watermap



(Fabian Keil)

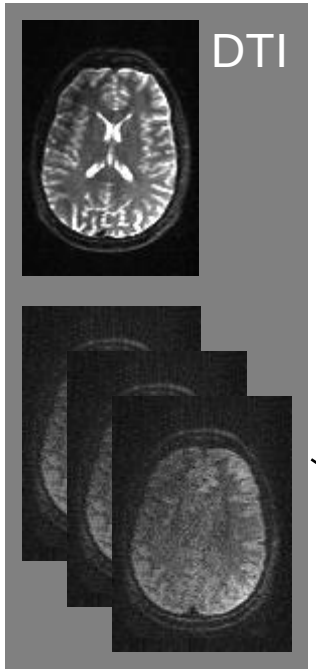
Examples: Perfusion/ Vessels (Angiography)

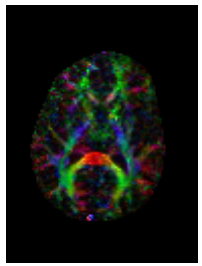
Examples: Metabolism (^{23}Na-Imaging)

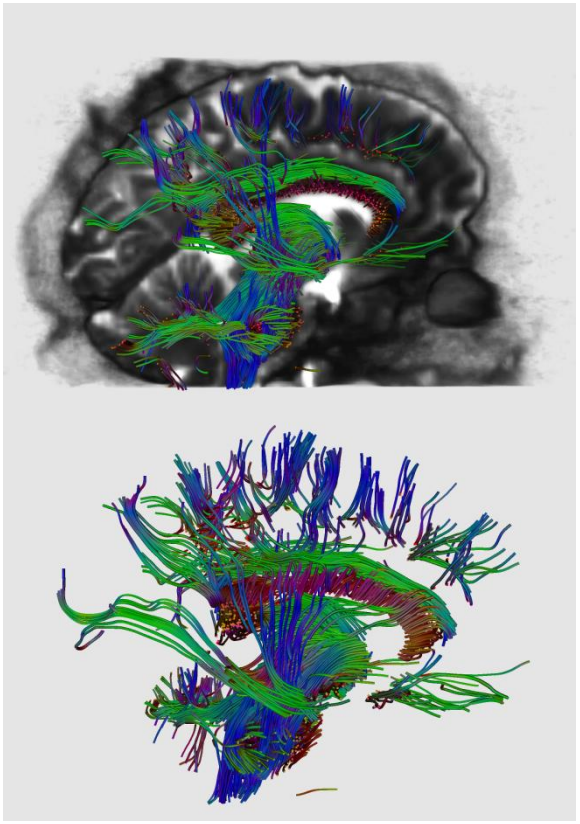


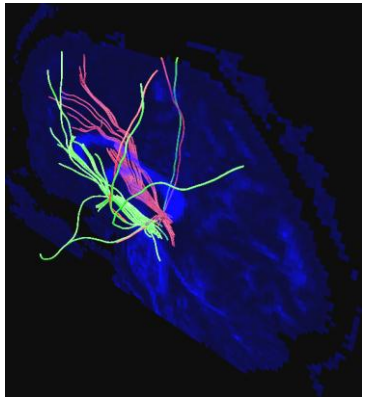
(Sandro Romanzetti)

Examples: Connectivity/ Fibres (DTI)









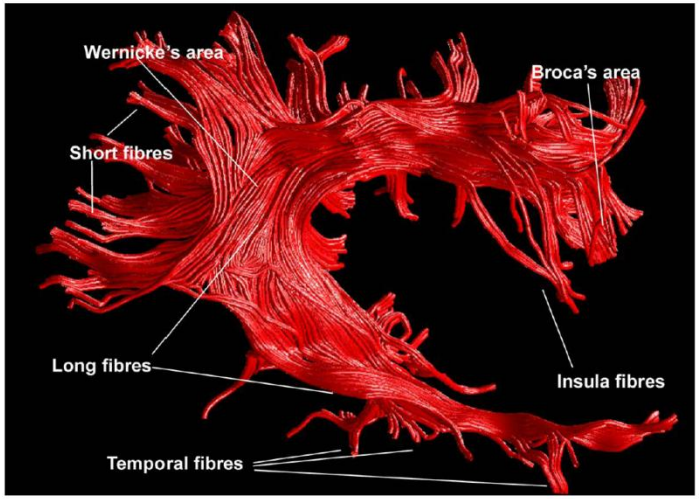
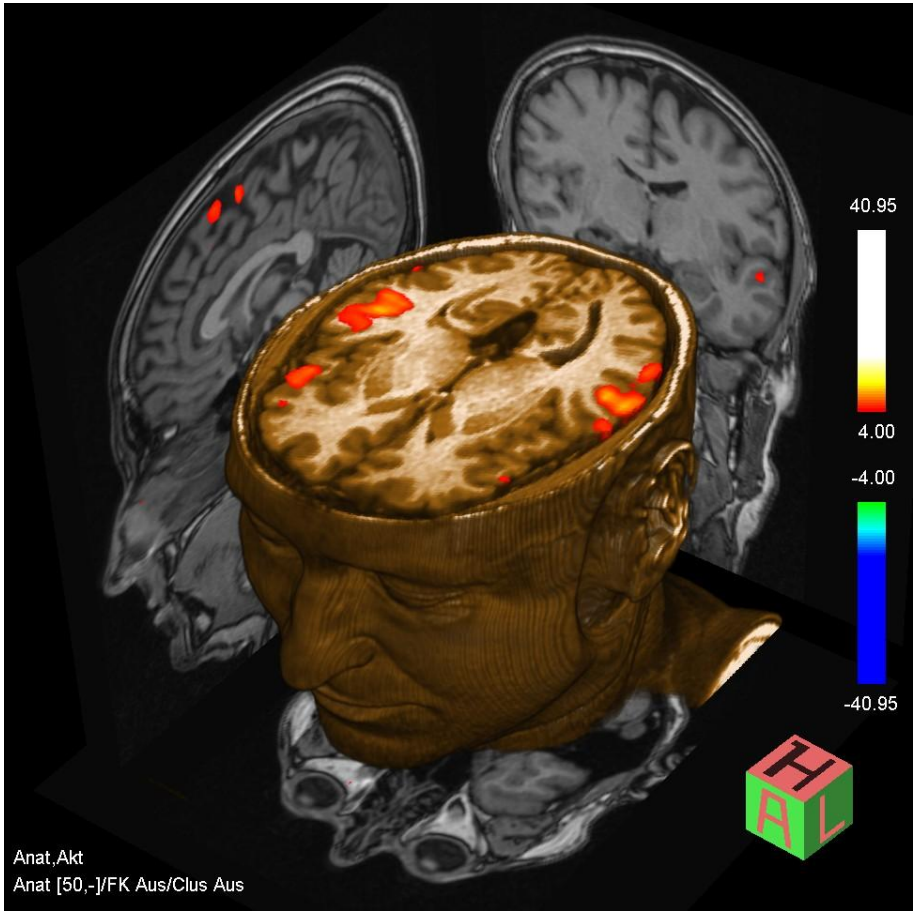


Figure 1. Streamline reconstruction of the superior longitudinal fasciculus (image courtesy Derek Jones).

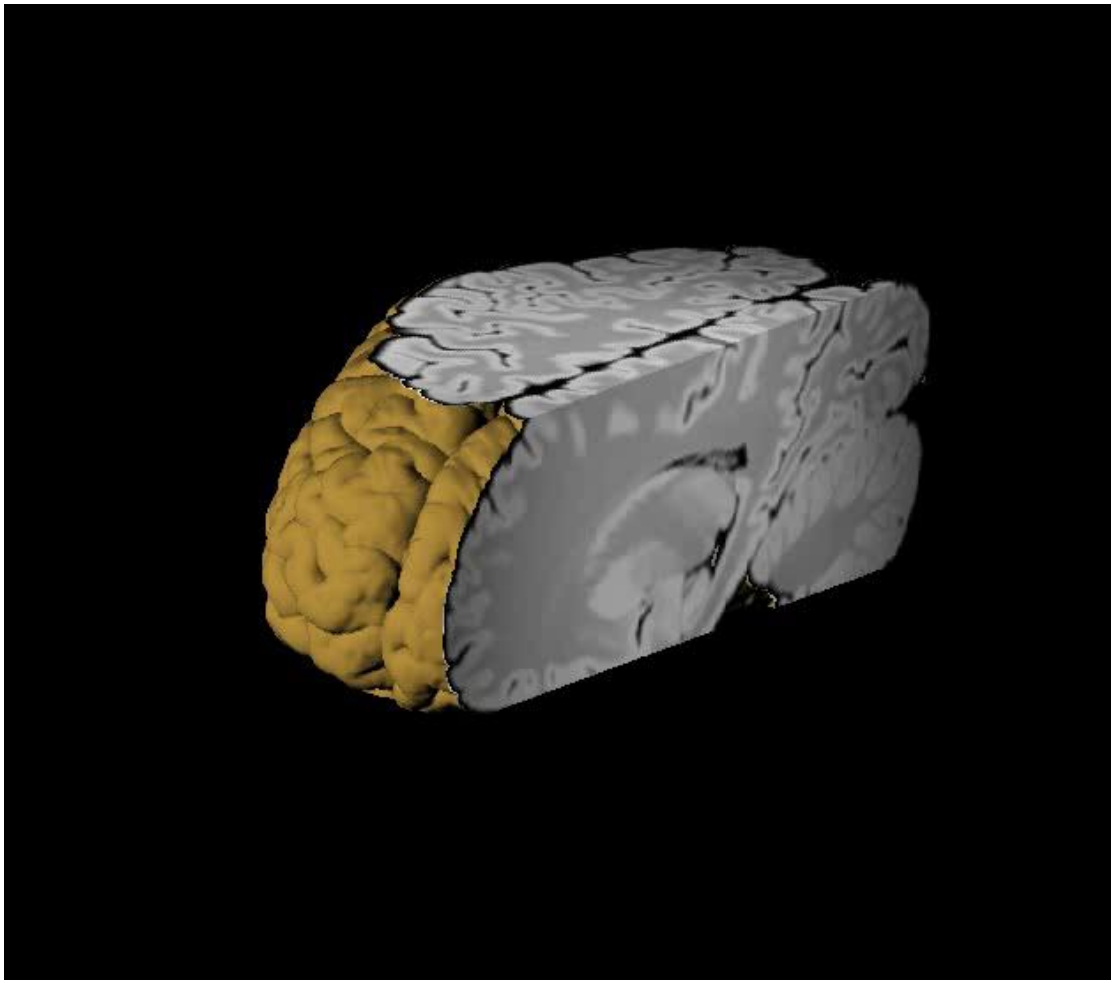
Rüdiger Stirnberg

Examples: Function/ Activity (fMRI)



http://www.radiologie.ruhr-uni-bochum.de/imperia/md/images/institut/mrt/fmri.jpg

Examples: inner volume excitation



Kaveh Vahedipour

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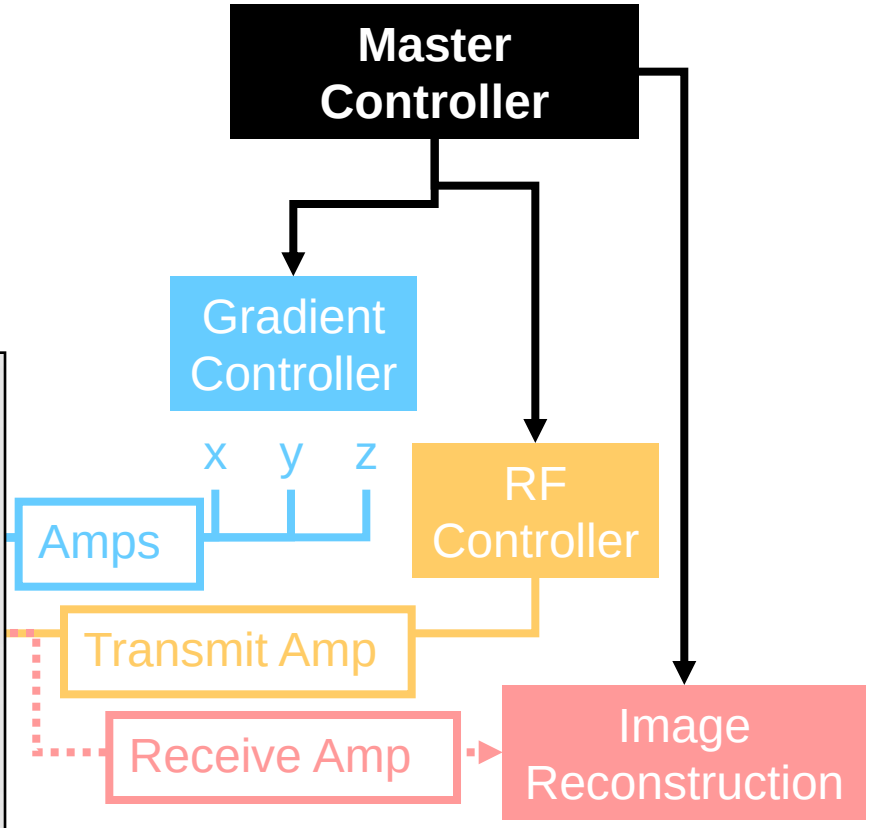
MR Scanner Components (simplified)

3 Types of magnetic fields:

1. Static field ($B_0 \parallel z$)
2. Dynamic gradients ($G_i = dB_z/dr_i$)
3. Radio-frequency ($B_1 \perp z$)



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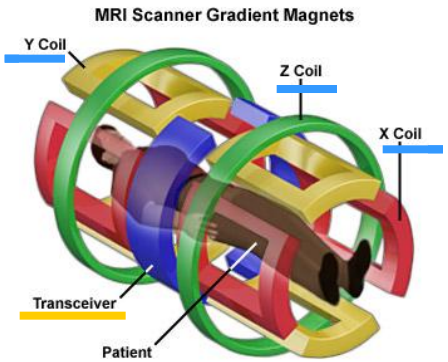

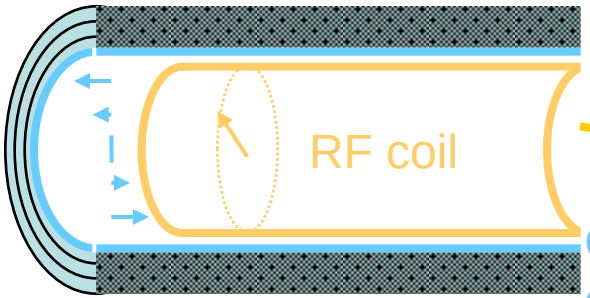

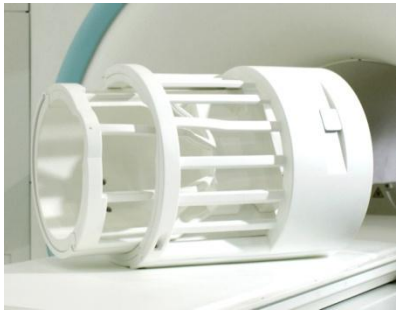

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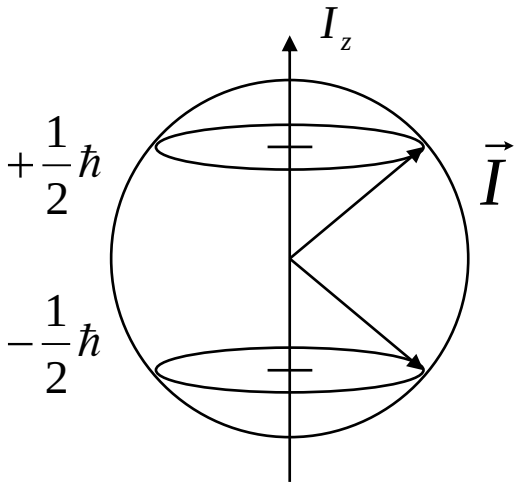
Nuclear Spin – Basic overview

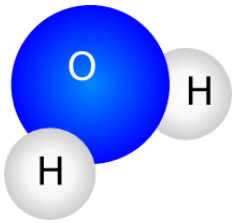
- Atomic nuclei are composed of *protons* and *neutrons* that have the quantum mechanical (QM) property of a ***spin***.
- Unlike the classical angular momentum the spin is an intrinsic property of elementary particles.
- The overall spin of the composed nucleus is called ***nuclear spin***.
- The possible values of the spins' *z*-component are quantized in integer steps.



Spin ½ example

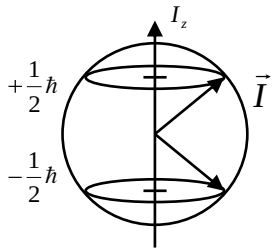
$2\pi\hbar = h$ = Planck's constant ≈ $6.626068 \cdot 10^{-34}$ J s

The Proton and Spin 1/2



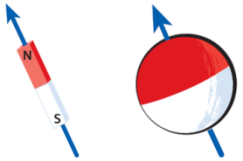
- The most simple atom is Hydrogen, H.
- As the human body consists of approx. 70% water, hydrogen is very prevalent.

- Hydrogen's nucleus is only one proton.
  Its nuclear spin quantum number is *S*=½, i.e. $I_z = \pm 1/2\,\hbar$



- Associated to every nuclear spin is a **nuclear magnetic moment**, $\vec{\mu} = \gamma\vec{I}$

- γ is called the ***gyromagnetic ratio*** and is nucleus-specific.
  For hydrogen, γ = 267.513·10^6 rad/(s·T).

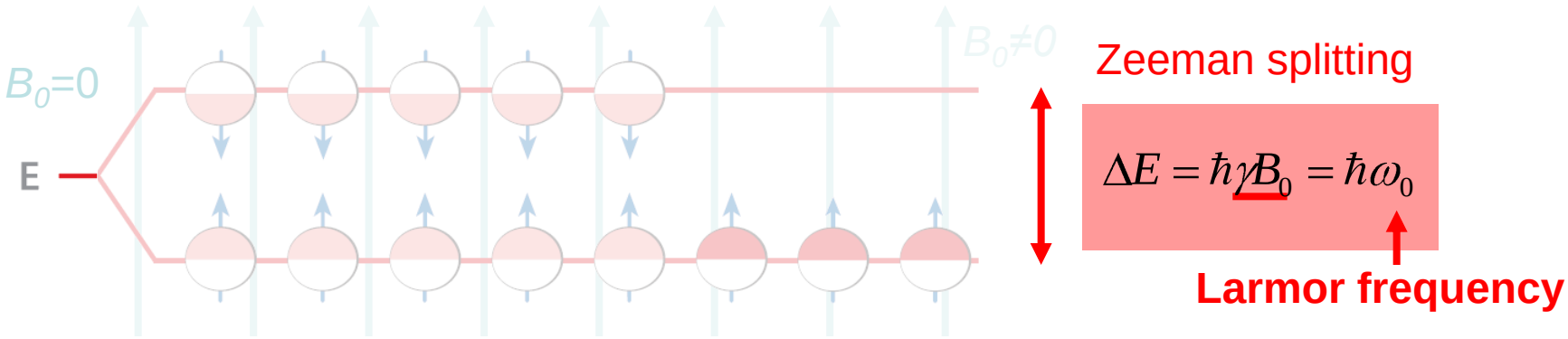


$\gamma / 2\pi \approx$ 42.6 MHz/T

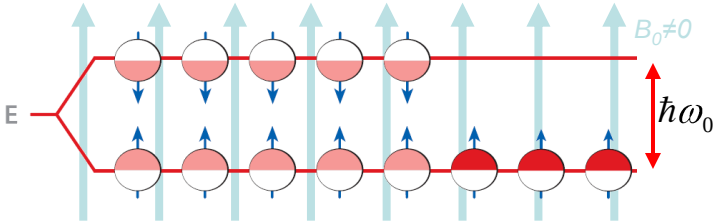
Nuclear Zeeman Effect

- Exposed to a static external magnetic field, B_0, the *z*-components of the nuclear magnetic moments align with B_0.
- Without loss of generality, the direction of B_0 defines the z-axis.
- Two possible orientations of the magnetic moments: parallel and anti-parallel with energies

$$E_{\uparrow,\downarrow} = \mu_{z_{\uparrow,\downarrow}} B_0 = \mp \frac{1}{2} \hbar \gamma B_0$$



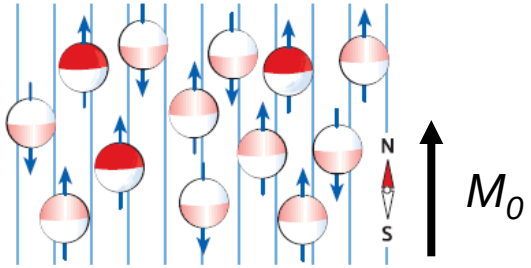
Equilibrium Magnetisation



- There is a small excess of spins in the lower energy states.
- The population fraction is given by **Boltzmann statistics** $\frac{n_\downarrow}{n_\uparrow} = e^{-\Delta E / k_B T}$ which is a very tiny number at room temperature *T*.

- However, the small excess amounts in a **measurable macroscopic magnetisation**, *M*, which is the ensemble average of the nuclear magnetic moments.

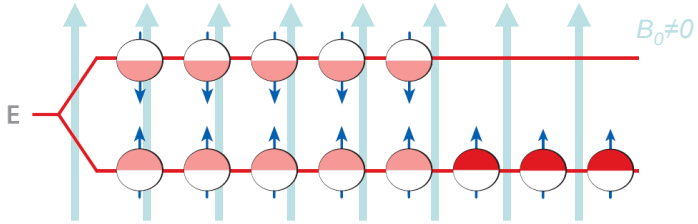
- The equilibrium magnetisation for *N* atoms with nuclear spin quantum number, *S*, is given by Curie's law:



$$M_0 = N \frac{\gamma^2 \hbar^2 S(S+1)}{3 k_B T} B_0$$

k_B = Boltzmann constant ≈ $1.381 \cdot 10^{-23}$ J / K

Equilibrium Magnetisation – Example



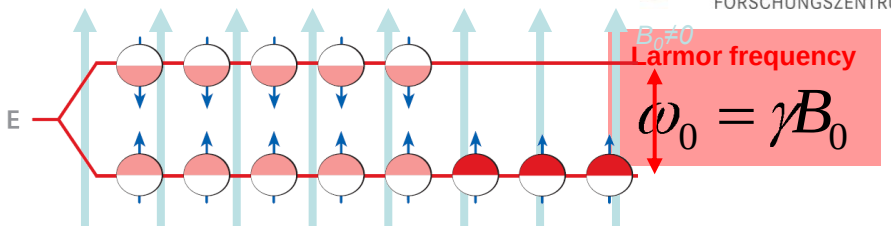
$$\text{Boltzmann: } \frac{n_\downarrow}{n_\uparrow} = \exp\left(-\frac{\Delta E}{k_B T}\right),\ k_B = 1.381 \cdot 10^{-23}\,\text{J/K}$$

- At B_0=1.5T and body temperature, T=310K, the spin ***excess*** in the lower energy state amounts only

$$\begin{aligned}\left(\frac{n_\uparrow}{n_\downarrow} - 1\right)_{1.5\,\text{T}} &= e^{+\Delta E(B_0 = 1.5\,\text{T})/k_B T} - 1 \\ &= \exp\left(\frac{4.1857 \cdot 10^{-26}\,\text{J}}{1.381 \cdot 10^{-23}\,\text{J/K} \cdot 310\,\text{K}}\right) - 1 \\ &= \exp(9.7772 \cdot 10^{-6}) - 1 \approx 0.0000097772\end{aligned}$$

- Since for small x: exp(x) ~ 1 + x, the spin excess in the lower energy state is proportional to the B_0 field strength.

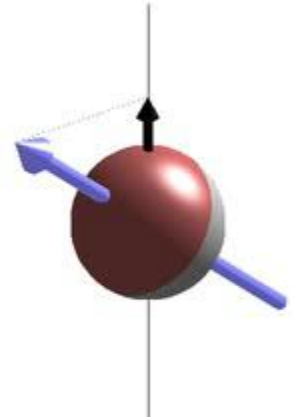
Larmor Precession



- Recall that in the equilibrium only the spin's *z*-components are aligned with B_0.
- The transverse components perform circular motion

➔ All **spins precess about B_0 with the Larmor frequency, ω_0!**

➔ But the resulting **magnetisation is static and aligned with B_0!**



- **Nuclear Magnetic Resonance**: an EM wave oscillating perpendicular to B_0 at Larmor frequency deflects the equilibrium alignment!

➔ **the magnetisation precesses about B_0 with the Larmor frequency, ω_0!**

Larmor Precession – Example

$$\gamma = 267.5 \cdot 10^6 \,\mathrm{rad/sT} \text{ and } \omega_0 = \gamma B_0$$

- At B_0=1.5T clinical field strength the Larmor frequency (angular frequency) is

$$\begin{aligned} \omega_0(B_0 = 1.5\,\mathrm{T}) &= 267.5 \cdot 10^6 \,\mathrm{rad/sT} \cdot 1.5\,\mathrm{T} \\ &= \underline{\underline{401.25 \cdot 10^6 \,\mathrm{rad/s}}} \end{aligned}$$

- In ordinary frequencies, this corresponds to **f_0=63.9MHz**.
- Radiofrequencies!

- For 9.4T human research scanner: f_0=400MHz!

Descriptive Derivation of the Larmor Precession

- The magnetic field causes a torque, ***T***, on a magnetic moment, ***μ***.

$$\vec{T} = \vec{\mu} \times \vec{B}$$

- The torque is the 1st derivative of the angular momentum, ***I***, with respect to time.

$$\vec{T} = \frac{d\vec{I}}{dt} = \frac{1}{\gamma}\frac{d\vec{\mu}}{dt}$$

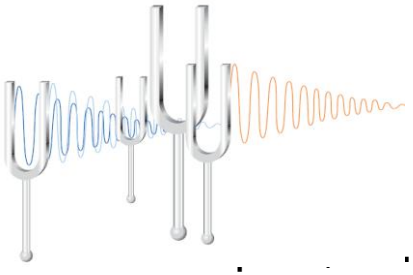
- Ensemble averaging from the microscopic magnetic moments to the macroscopic magnetisation:

$$\boxed{\frac{d\vec{M}}{dt} = \vec{M} \times \gamma\vec{B} = \vec{\omega} \times \vec{M}}$$

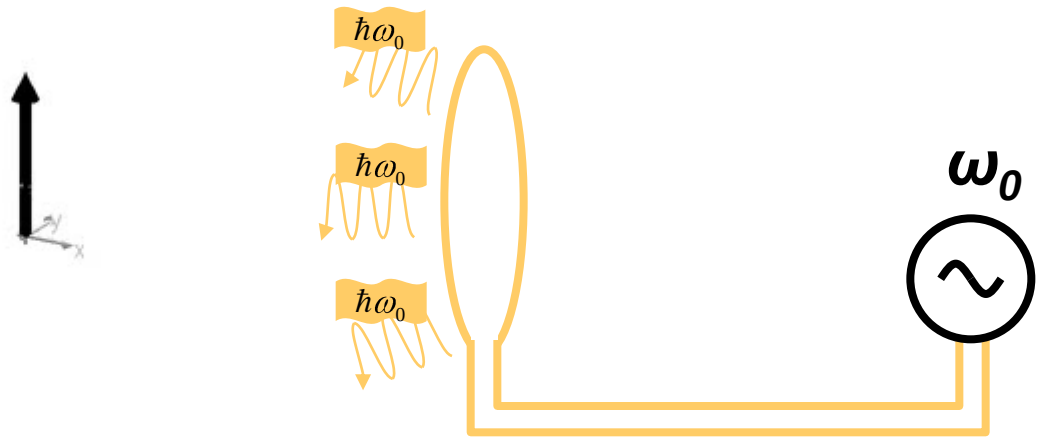
Equation of motion* solved by precessional motion with frequency **ω**=-γ***B***. (clockwise).

* “Bloch-Equation without relaxation”

Excitation



- Apply a **resonant radiofrequency (RF) pulse** to excite the macroscopic magnetisation from its equilibrium.
- **Resonance** ↔ frequency of RF pulse = Larmor frequency $\boldsymbol{\omega_0}$.



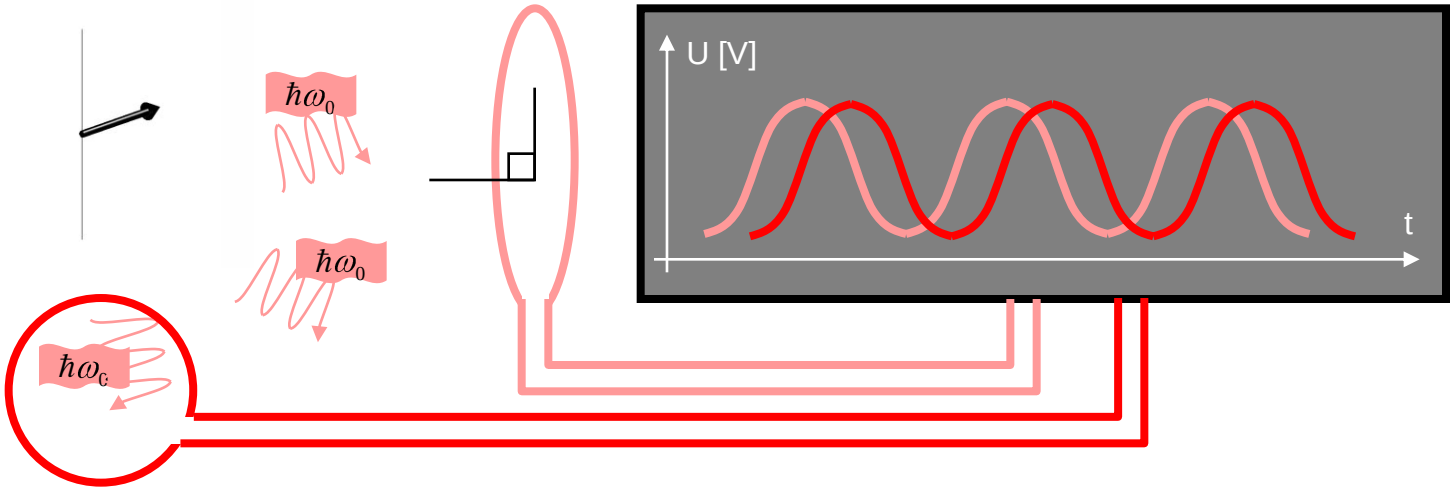
MR Signal Reception

- Faraday's law:

  "A changing magnetic flux induces a voltage in a conductive loop."

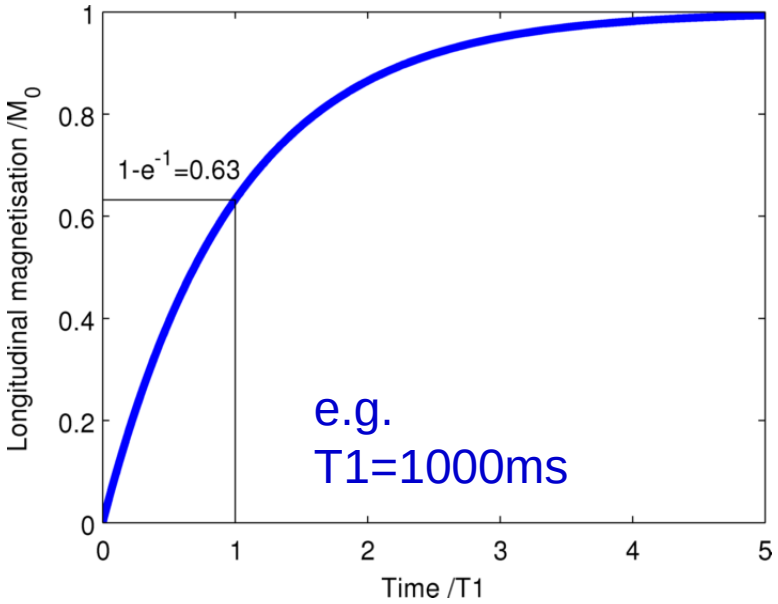
  (electromotors, dynamos , …)

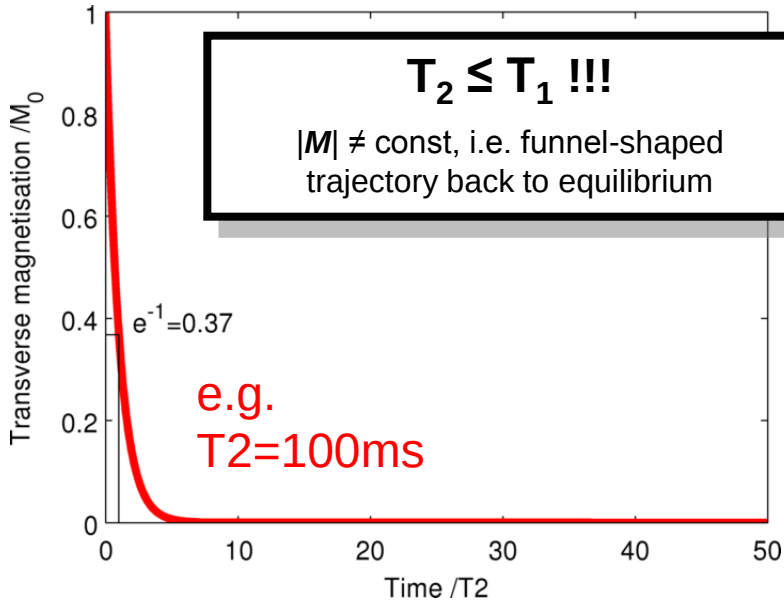
- Maximise the area of the loop by a orienting it perpendicular to the transverse plane (in-plane would cause no voltage at all)



Relaxation

- It is intuitive that equilibrium-magnetisation must be regained somehow.
- Heuristically described by two relaxation times:
  - T_1 (spin-lattice relaxation time) $M_z = M_0\left(1 - e^{-t/T_1}\right)$
  - T_2 (spin-spin relaxation time) $M_{xy} = M_0 e^{-t/T_2}$





$T_2 \leq T_1$!!!

$|\boldsymbol{M}| \neq$ const, i.e. funnel-shaped trajectory back to equilibrium

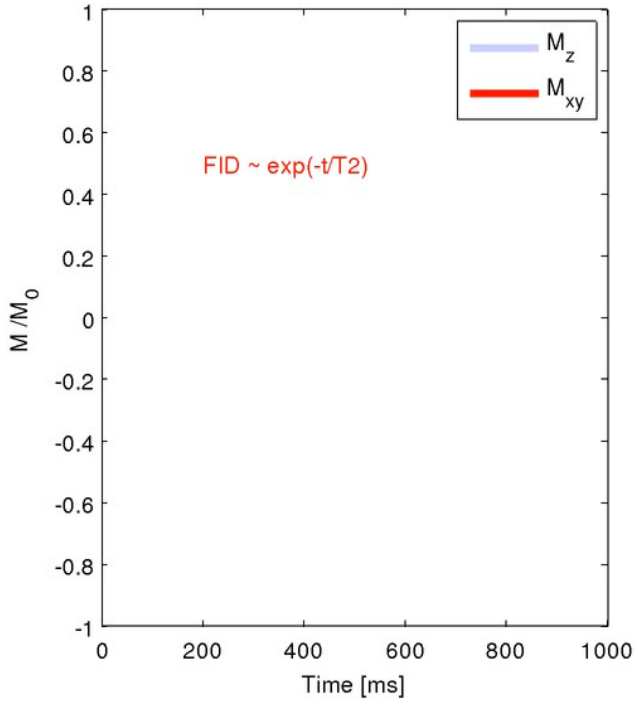
Classic Treatment of Spin ½ MR: Bloch Equation with Relaxation

- Including the relaxation terms, the equation of motion is given by the ***phenomenological Bloch equation***:

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} - \frac{M_x \hat{e}_x + M_y \hat{e}_y}{T_2} - \frac{(M_z - M_0)\hat{e}_z}{T_1}$$

Free Induction Decay (FID)

- The received MR signal decays exponentially with the time constant T_2.



The MR Signal Equation

The ***transverse magnetisation***, $M_{xy}(\vec{r},t) = M_x(\vec{r},t) + iM_y(\vec{r},t)$ of all magnetization vectors induces the voltage:

$$S(t) \propto \int_{Coil\ Volume} M_{xy}(\vec{r},t)\, d^3r$$

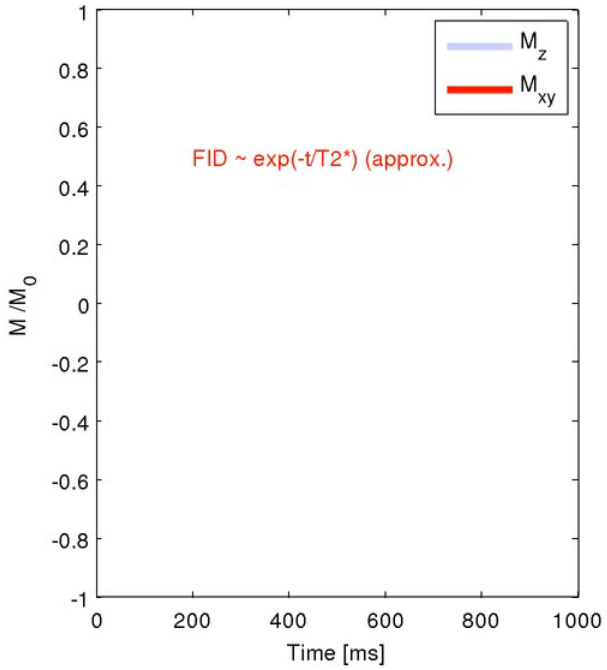
The ***phase coherence*** of $M_{xy}(\vec{r},t) \equiv A(\vec{r},t)e^{i\varphi(\vec{r},t)}$ determines the signal amplitude:

- $\varphi(\mathbf{r},t_0)$ = const $\forall$ $\mathbf{r} \in$ V $\Rightarrow$ $|S(t_0)|$ = max
- $\varphi(\mathbf{r},t_0)$ uniquely distributed on [0, 2π] $\Rightarrow$ $|S(t_0)| \approx 0$

Free Induction Decay (FID), Part II

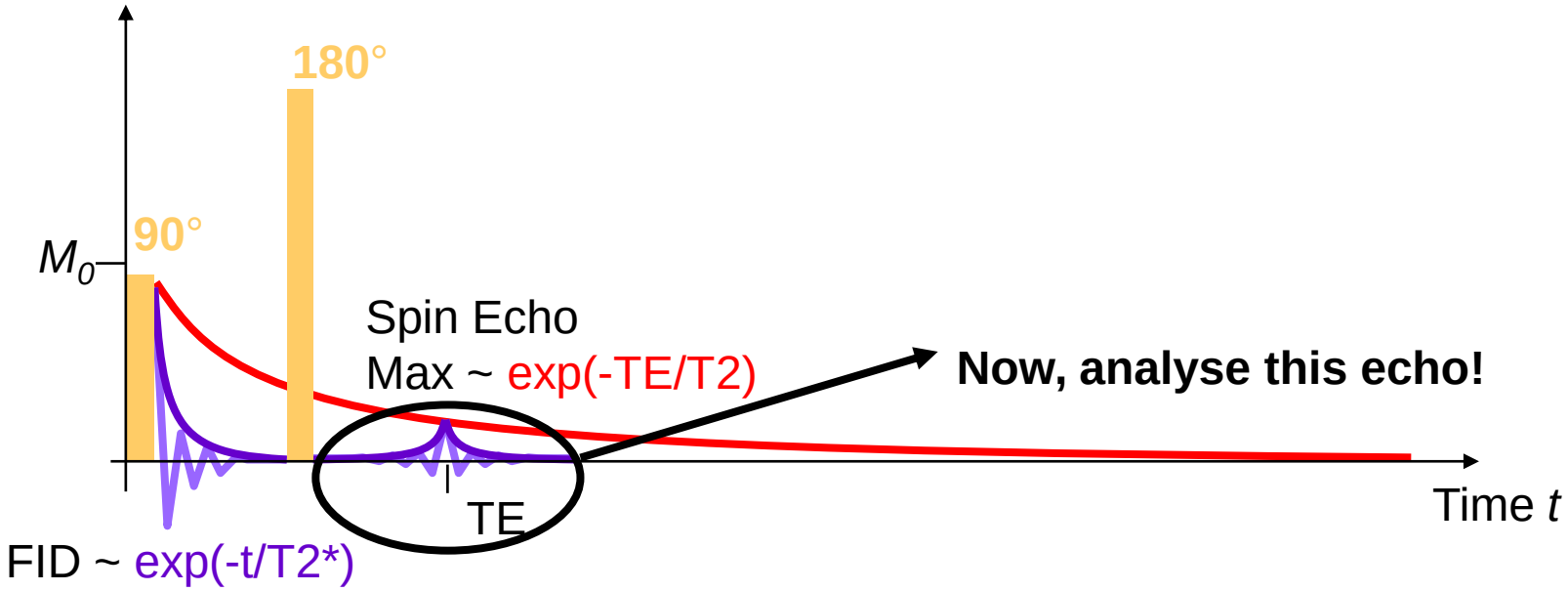
$T_2^* < T_2$!!!

- The received MR signal decays exponentially with the time constant T_2.
- Local B_0 inhomogeneities cause an even ***shorter effective decay time, T_2^**** ("T-two-star") through *loss of phase coherence*:



Spin Echo (SE)

- Signal losses due to B_0 inhomogeneities is reversible!
- Recall a ***spin echo*** by inversion of the spins using a 180° pulse.
- The individual spin's phase evolution due to the B_0 induced *frequency dispersion (T_2*)* is "rewound". But T_2 *decay is irreversible*.



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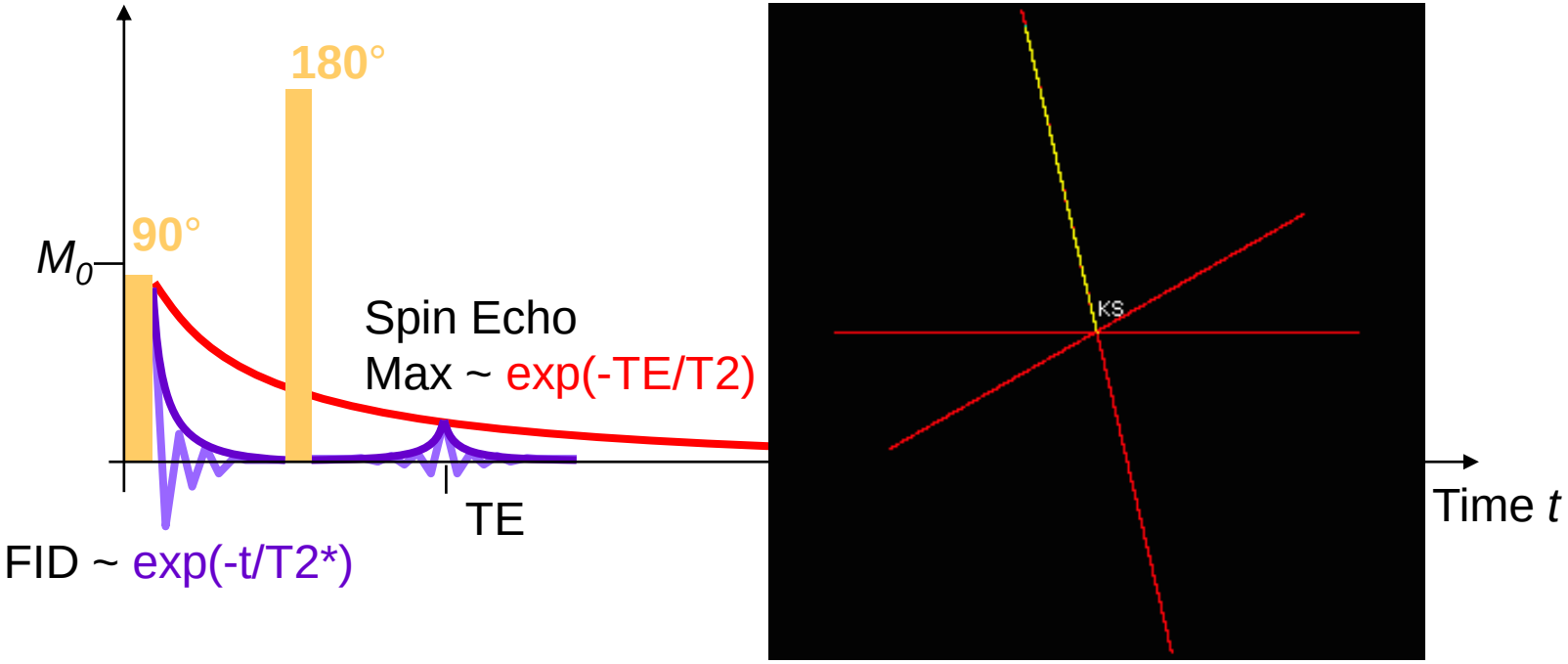
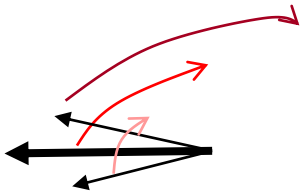
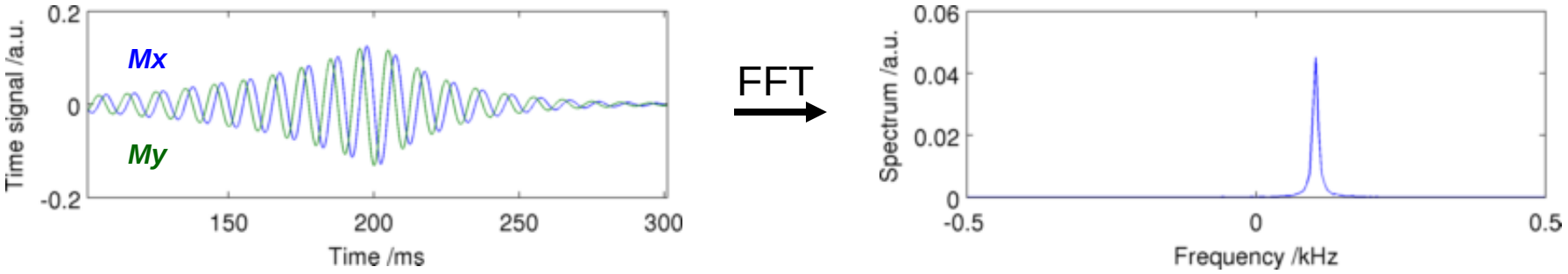


Fourier Transformation (FT)



- We expect a frequency dispersion of subgroups of spins – so called ***isochromats***.
- That means: most spins precess with the Larmor frequency but some are a little faster and some a little slower.

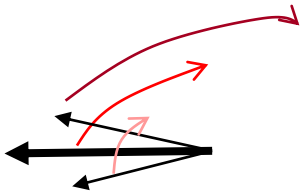
- Obtain spectrum by a ***Fourier transform (FT)*** of the time signal, usually a Fast Fourier Transform (performant implementation of a discrete FT)



Note: the 0.1 kHz proton peak shown in this simplified example would correspond to a very low field strength of $B_0 \approx 2.3\mu T$ according to $f_0 = \omega / 2\pi = \gamma B_0 / 2\pi$.

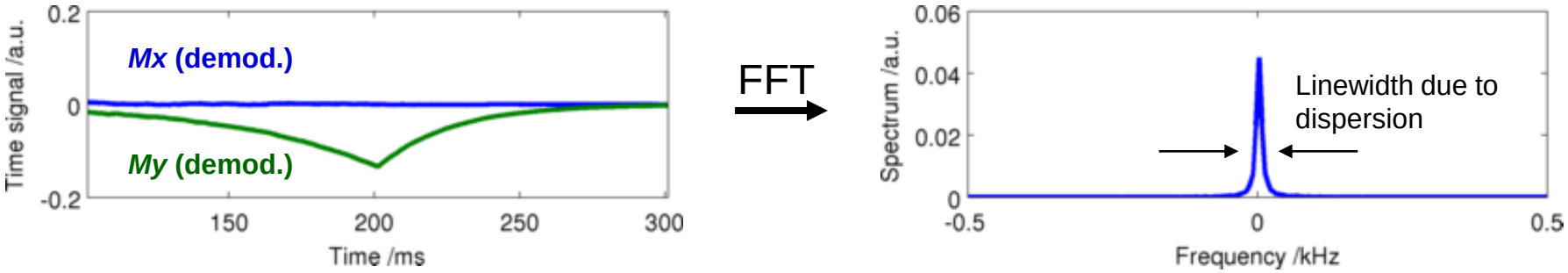
In clinical scanners, the proton frequency ranges between 63.9MHz at 1.5T and 127.7MHz at 3T.

Fast Fourier Transformation (FFT)



- The signal is always demodulated in an analog mixing step before analog to digital conversion (ADC, i.e. digitised sampling of the MR signal).

- This demodulation corresponds to moving to another frame of reference, the “rotating frame”.

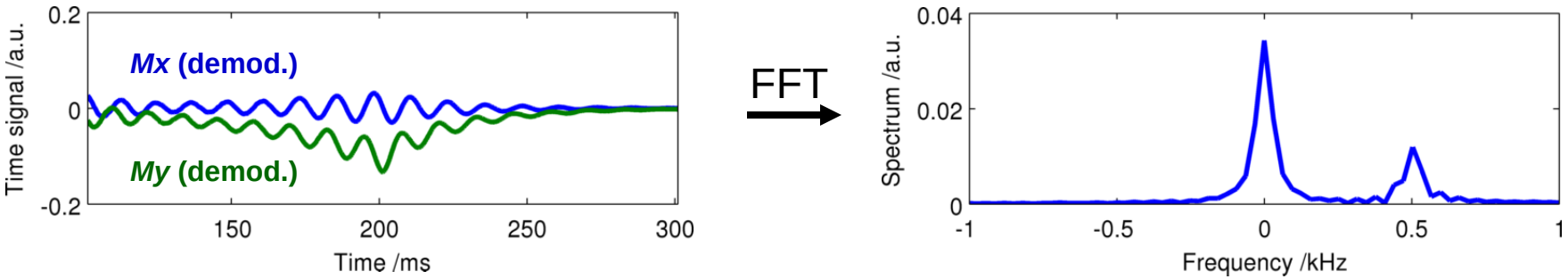


- In this frame of reference (rotating with the Larmor frequency about z), all resonant spins point to where they were deflected initially.

NMR Spectroscopy

- Your probe may have protons with a slightly varied precessional frequency.
- I.e. the Larmor frequency is influenced by the chemical binding of the protons (“chemical shift”).

➔ Nuclear Magnetic Resonance (NMR) spectroscopy.



- Artificial example with water (Larmor frequency) and a compartment with a 500Hz chemical shift (Larmor frequency + 0.5kHz)

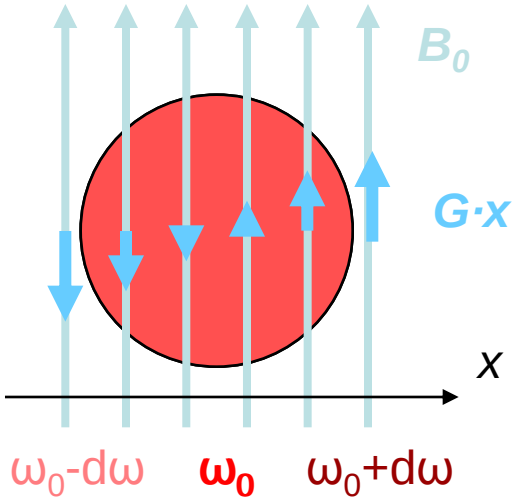
Spatial Encoding

- The basic idea of MRI:

  Make the precessional frequency a function of space!

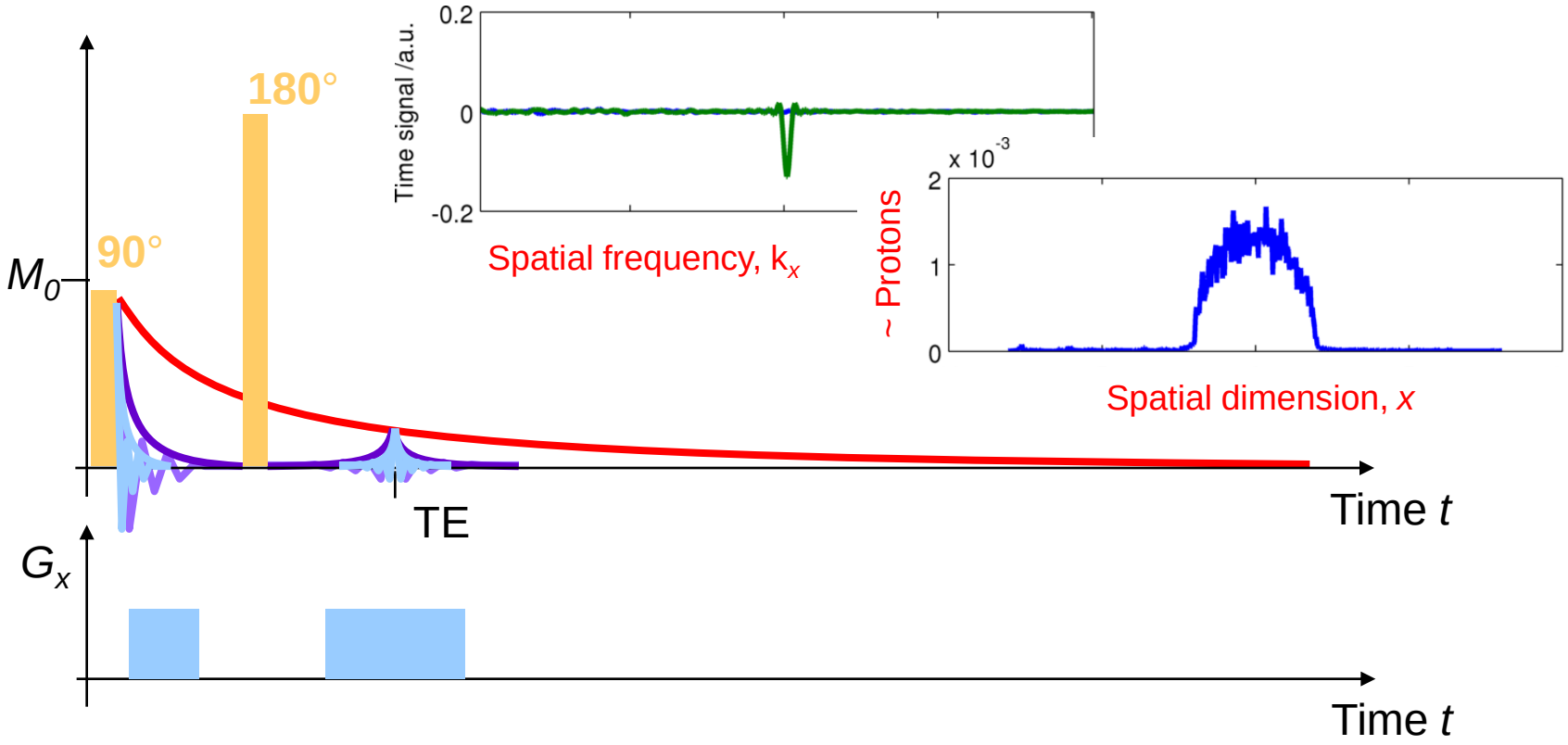
- The “spectrum” then reflects spatial distribution.

- Linear field gradients of the B-field in z-direction, e.g. $G_x = dB_z/dx$



Spin Echo with Frequency Encoding

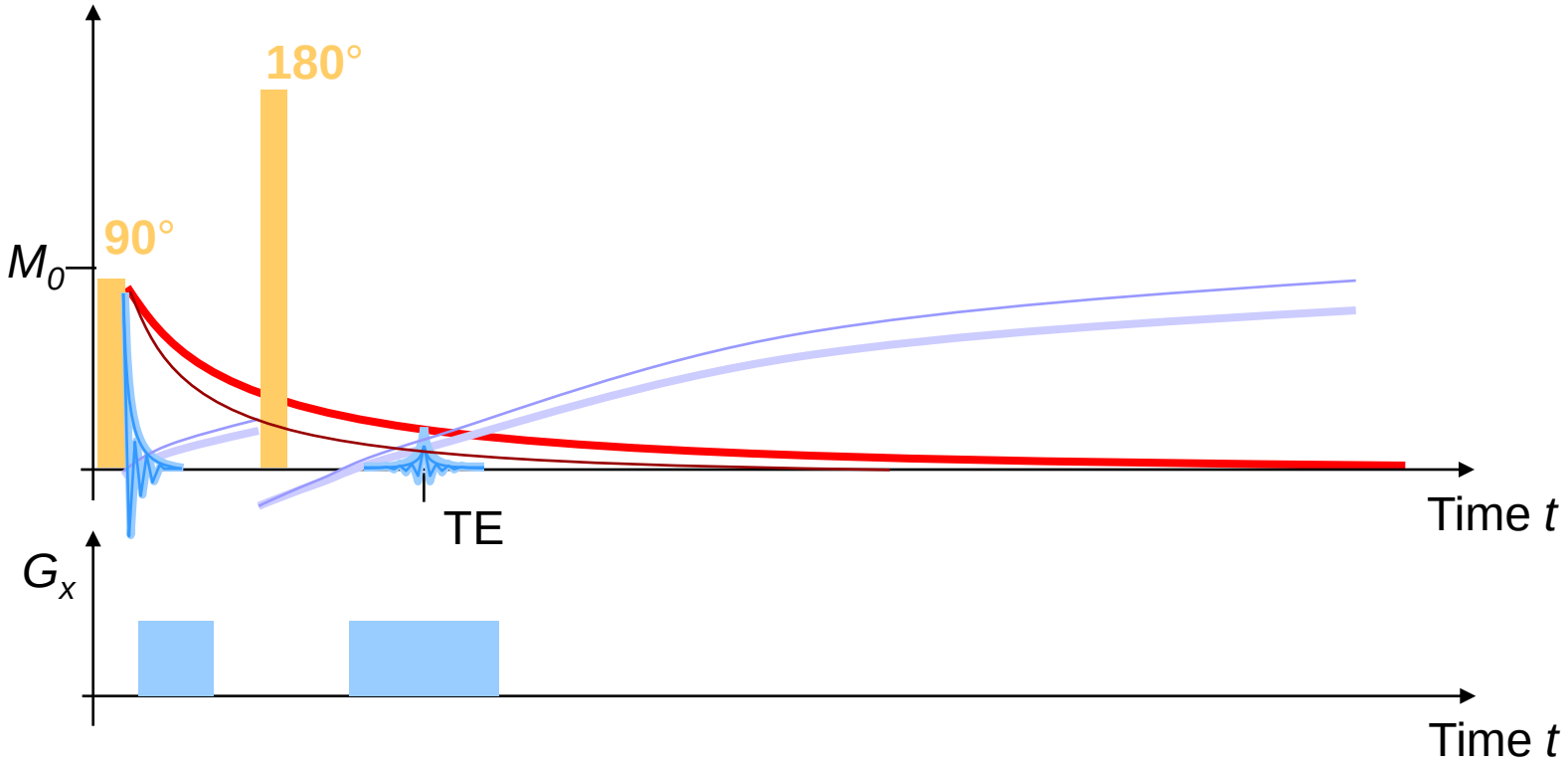
Take the spin echo and introduce gradients along *x*.



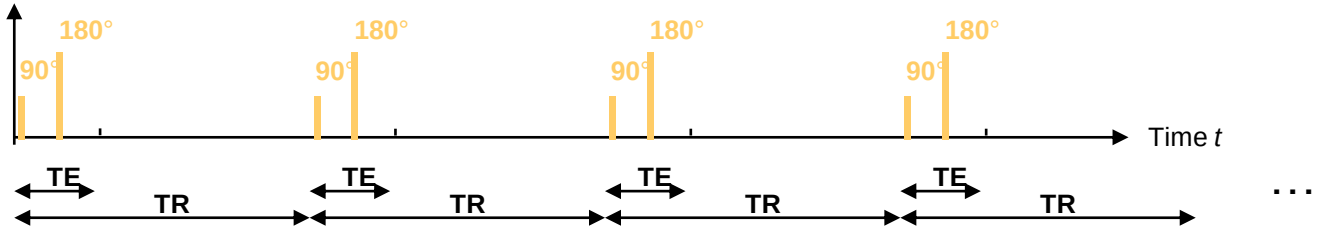
Contrast

Different tissues have different T1/T2 times

➔ Generate different contrasts through sequence timing parameters!

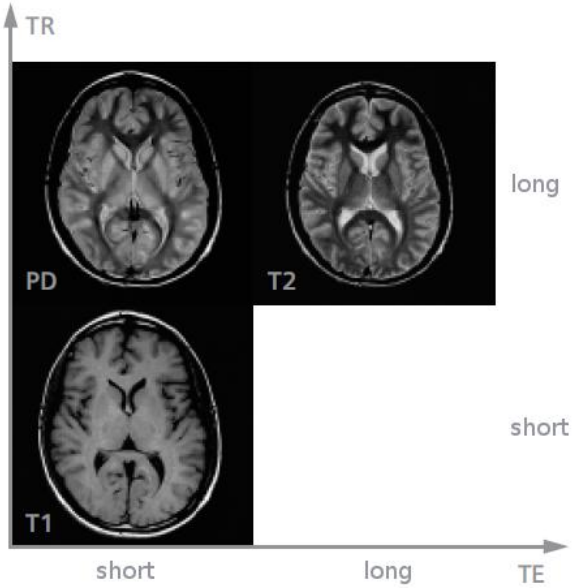


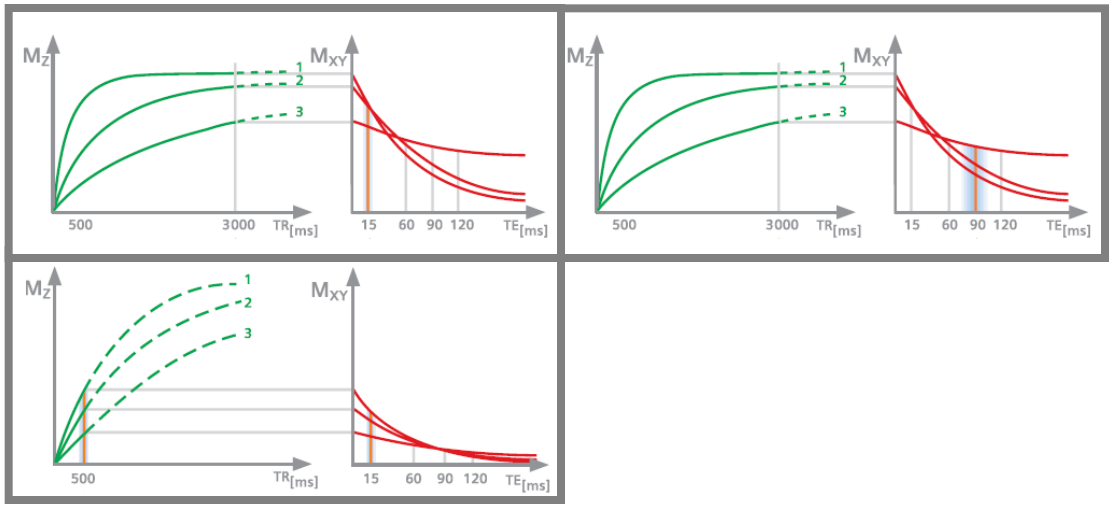
Contrast



Choice of

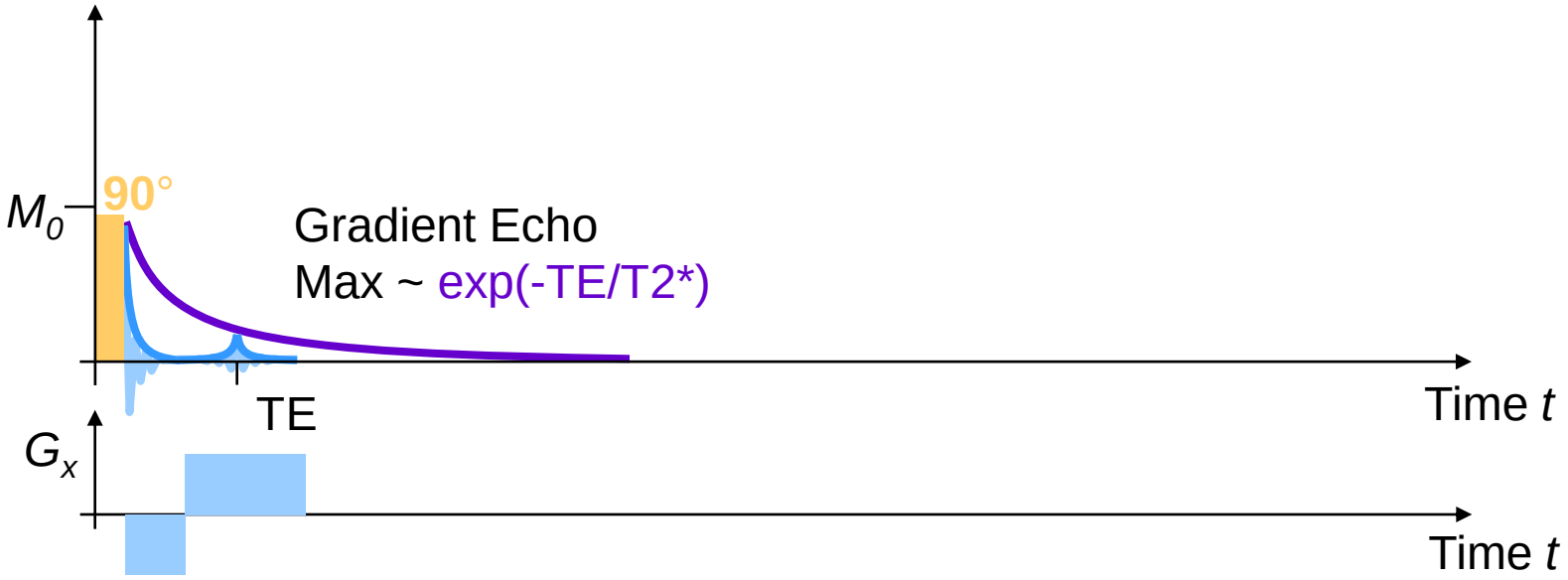
- Echo time (TE)
- Repetition time (TR)



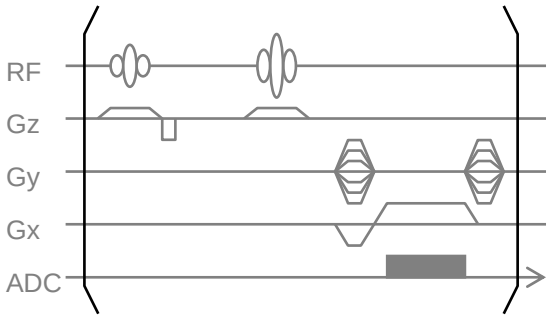


Gradient Recalled Echo (SE)

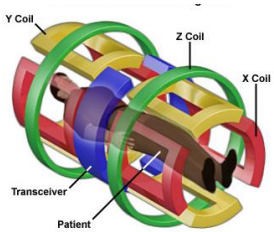
- Recall a ***gradient echo*** by *dephasing* and subsequent *rephasing* in the opposite direction.
- Much faster and less RF than spin echo.
- But: the individual spin's phase evolution due to the B_0 induced *frequency dispersion (T_2*)* is *NOT* "rewound".



Imaging Sequences



- Gradient encoding must be applied in 3 dimensions for a complete spatial encoding.
  - During signal readout: *frequency encoding*
  - In a step-wise procedure (for 1 or more dimensions): *phase encoding*
  - Very common, simultaneous with excitation: *slice-selection*



- Gradients in x, y, and z-direction (dB_z/dx, dB_z/dy, dB_z/dz)

- Complex pulse sequences are programmed to obtain all necessary data for a complete reconstruction of the subject. Often: loop-structure.

- General goal: fill the "*k*-space" most quickly.
  (image space ←FFT→ *k*-space = FT-coefficients = spatial frequencies).

Outline

- Introduction
  - Magnetic Resonance Imaging
  - Appetiser – some Examples
- MR Scanner Components
- Physical Introduction
  - Spin ½ in B_0
  - Larmor Precession
  - Resonance (excitation and signal reception)
  - Relaxation
  - FID, Echoes and Fourier Transform
  - Basic Idea of MR Imaging
- Basic Safety Issues
- Sum-up

Basic Safety Issues – The Main Field

- Permanently cooled-down (liquid Helium, ~ 4K) superconductor produces the high, static magnetic field B_0 ~ 0.1 – 3 Tesla (1T = 10000 Gauss).
- Compare: the Earth's magnetic field on its surface ~ 0.5 Gauss.

➔ Huge force on ferromagnetic material growing rapidly when approaching the magnet.

➔ Objects like a normal fire extinguisher, surgical instruments, tools, keys etc. can cause immense (live-threatening) damage.

➔ Credit cards, cell phones, pacemakers will not work anymore.

➔ Generally, currents are induced in conducting material by changing magnetic flux, e.g. in organs when entering the scanner too quickly (from 0 to B_0). Can cause dizziness.

Basic Safety Issues – Field Gradients

- Imaging gradients are ramped up and down repetitively during an imaging sequence.
- Max. Amplitude ~ 40 mT/m.

➔ Slew rate is limited to max. 200 mT/(m ms) in order not to stimulate the patient's peripheral nerves (muscle contraction in arms, back, …)

➔ Patients should not cross their hands in order not to create a conducting loop through their body.

➔ Again, ferro-magnetic parts are not allowed, they would move and cause live-threatening damage.

Basic Safety Issues – RF fields

- High frequency em-waves are emitted to the patient in the order of 100 MHz.

➔ This can lead to hot spots at metallic edges, e.g. cables in contact with the skin or MR-unsafe implants.

➔ Make-up containing metallic particles can heat (also tattoos!)

- For higher frequencies the wavelength of the radio-waves inside tissue approaches the extent of, e.g., the head.

➔ High amplitudes due to standing-wave effects.

➔ Amplitude and duration of RF must be controlled due to SAR (Specific Absorption Rate) constraints. SAR is an important issue, particularly at high field strengths because of the reduced wave length.

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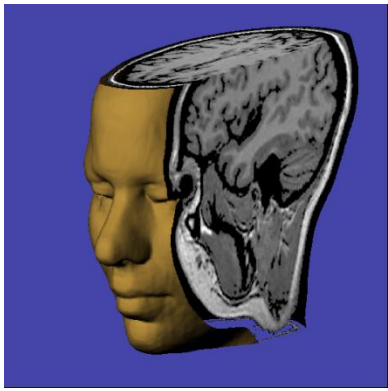
Sum-up

- Proton-spins in B_0: macroscopic magnetisation (along B_0 in equilibrium).
- Spins excitable using *resonant radio-frequency* pulses.
- Motion described by *Bloch-equation* (generally: Larmor precession).
- *Relaxation* described by T_1, T_2 (and T_2*).
- MRI acquires echo-signals, usually *Spin echoes* or *Gradient recalled echoes.*
- A general MRI sequence:
  - Excitation (RF pulse)
  - Encoding (gradients)
  - Data acquisition ➔ FFT ➔ image



Thank you for your attention

Introduction to MRI