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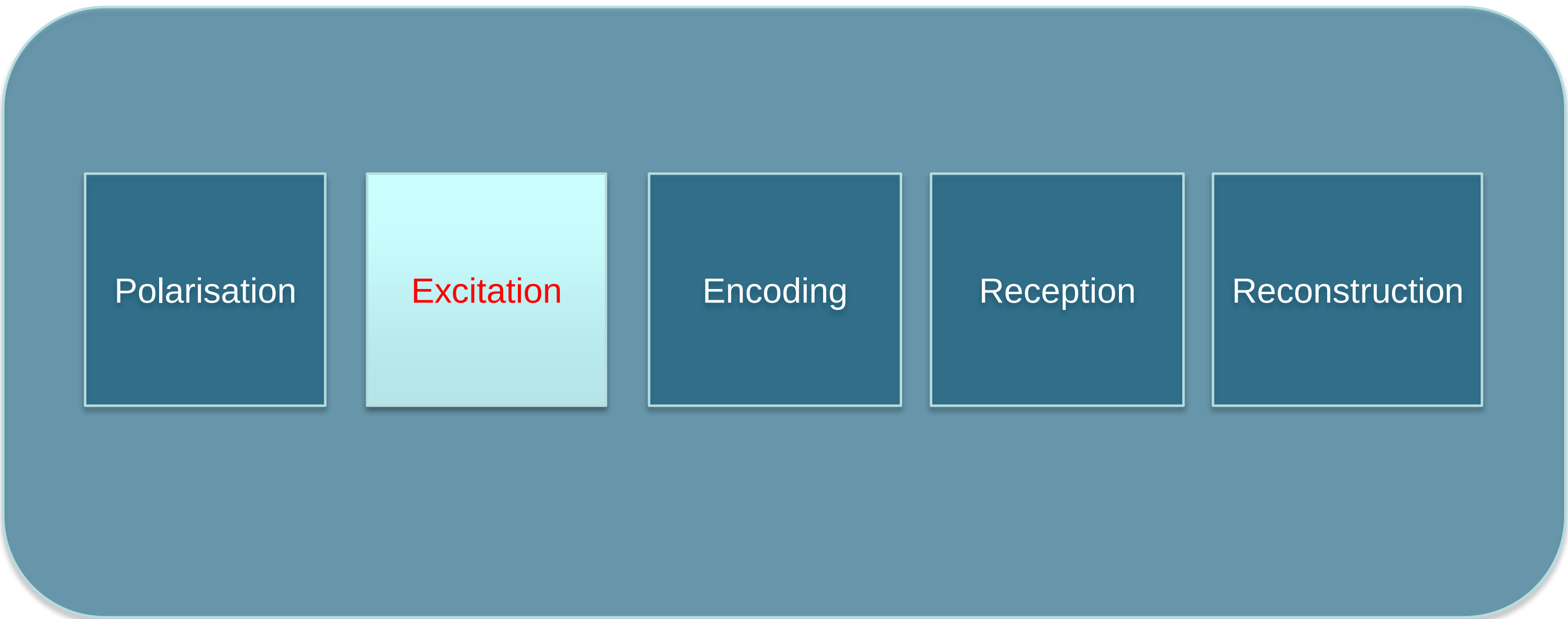
Magnetic Resonance Imaging

Excitation

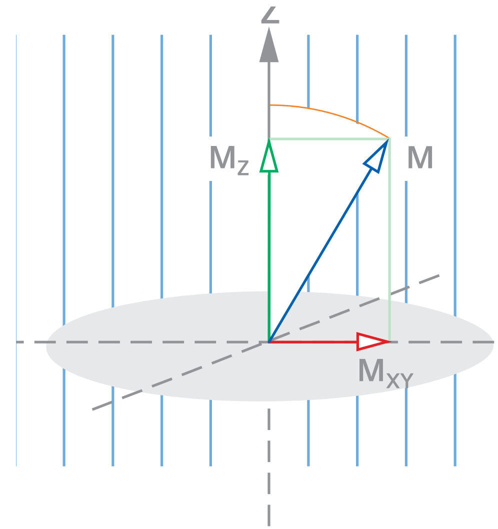
The Whole Picture

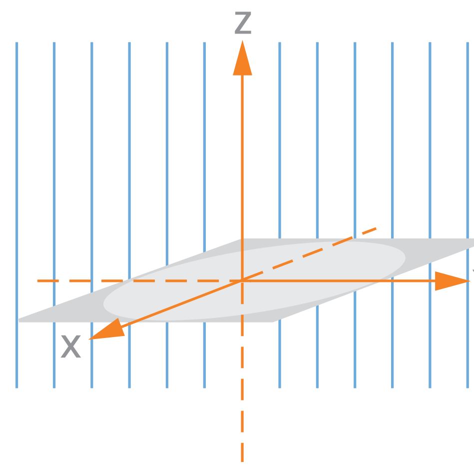


Steps of an MRI experiment

Reminder

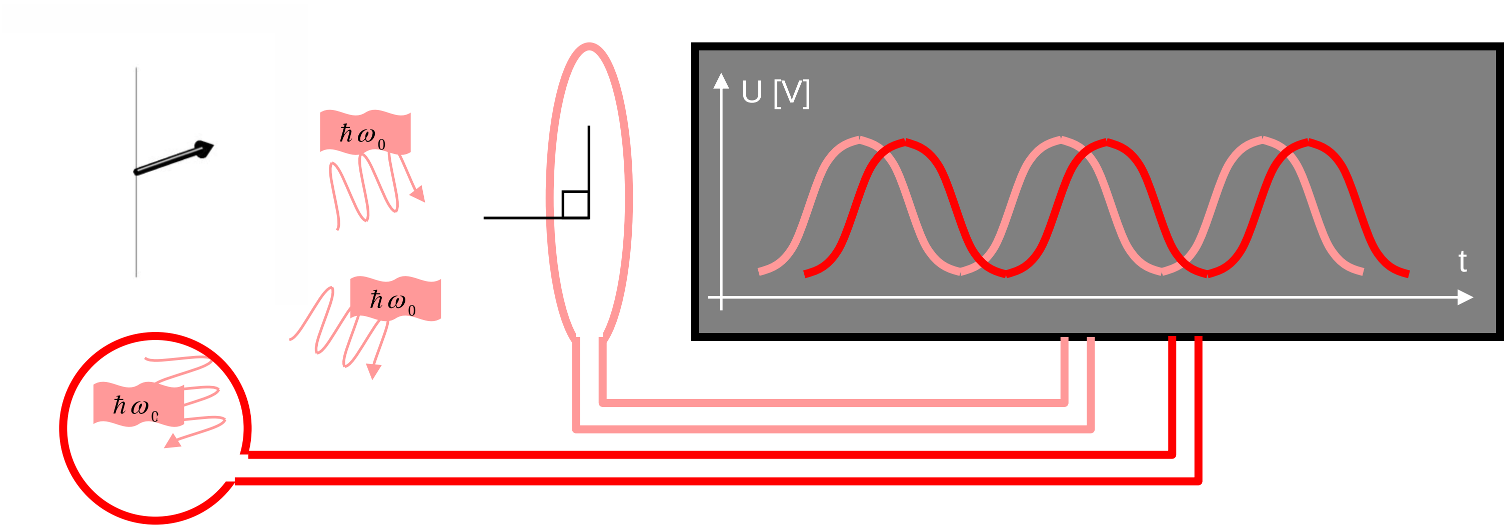
- Longitudinal axis – z
- Sample magnetisation M
  - Transverse magnetisation (M_x+iM_y)
  - Longitudinal magnetisation
- External field B
  - Static B_0 (z direction)
  - RF field B_1 (x-y plane)
  - Gradient fields $dB_z/d(x,y,z)$





Reminder: Reception of the FID signal

- Faraday's law: detect transverse magnetization $M_{xy}=M_x+iM_y$
- Signal Equation: $$S(t) \propto \int_{Coil\ Volume} M_{xy}(\vec{r},t)dV$$
- FID: exponential decay



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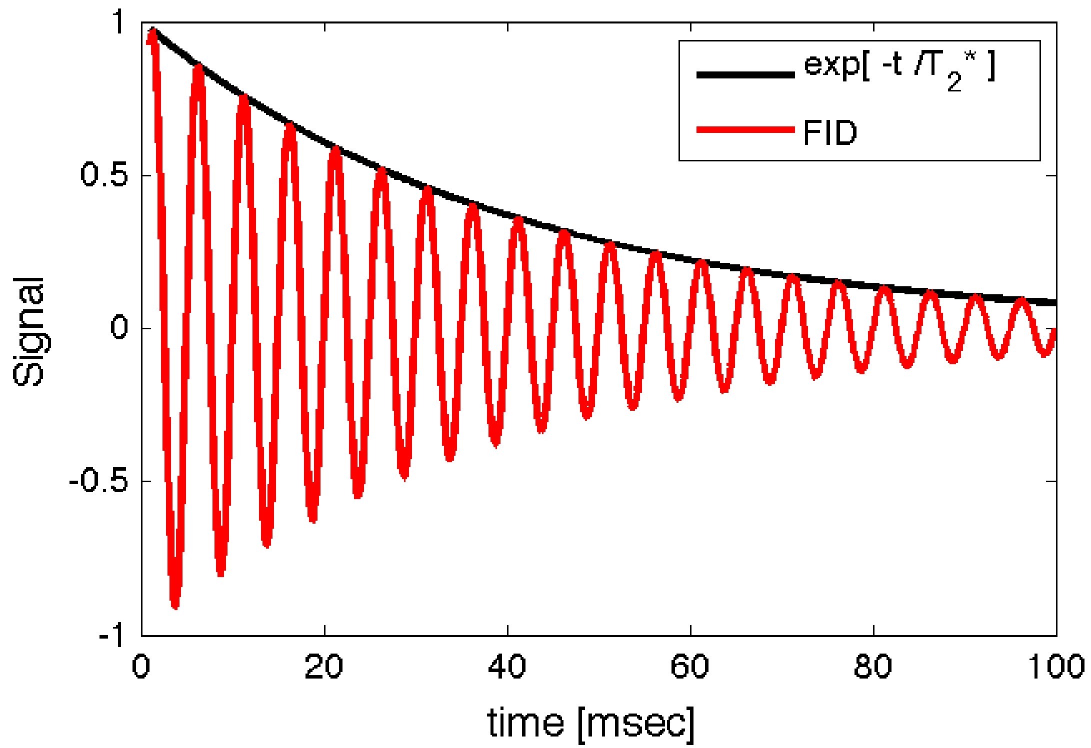


Outline

- Introduction
- Non-selective excitation
- Off resonance effects
- Selective excitation
- Safety Considerations - SAR

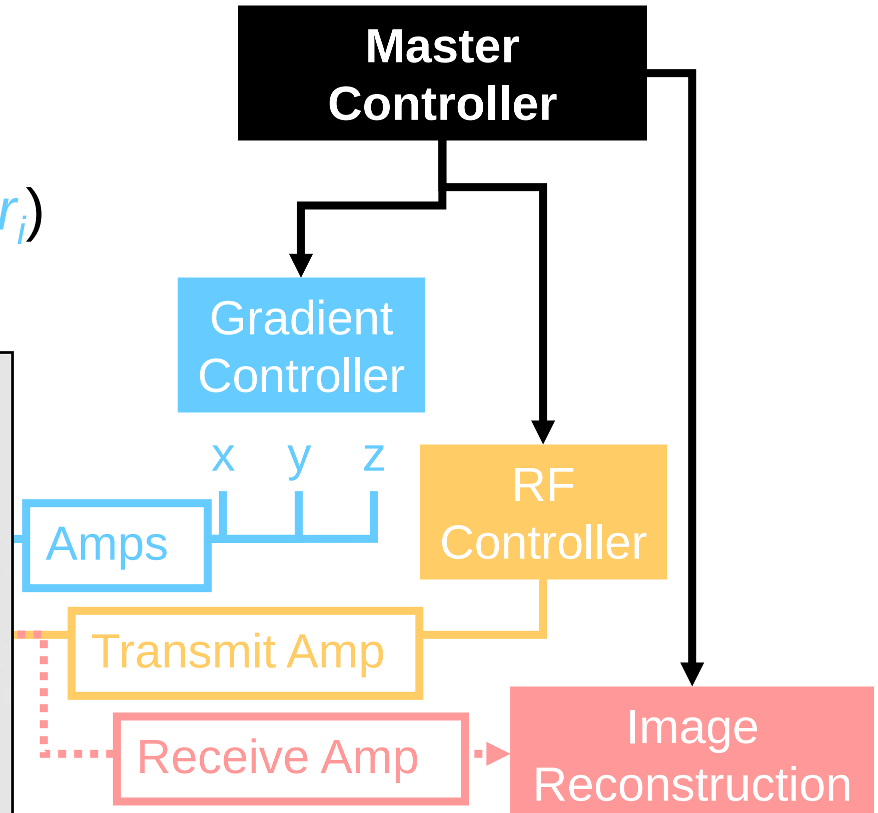
Introduction

- Excitation refers to the application of radio frequency (RF) pulses to generate transverse magnetisation
- Excitation is usually the first part of any MRI experiment
- Excitation is usually divided in
    - *Non-Selective Excitation*
    - *Selective Excitation*

MR Scanner Components (simplified)

3 Types of magnetic fields:

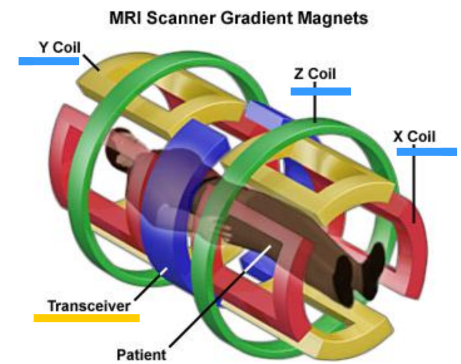
1. Static field ($B_0 \parallel z$)
2. Dynamic gradients ($G_i = dB_z/dr_i$)
3. **Radio-frequency** ($B_1 \perp z$)

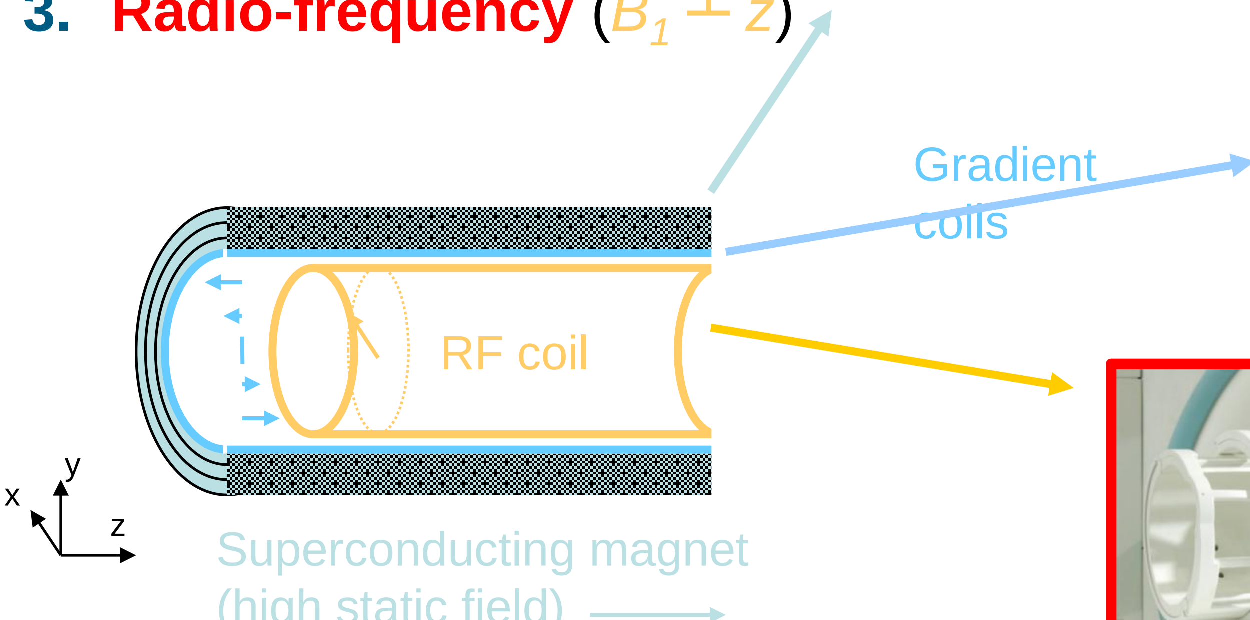


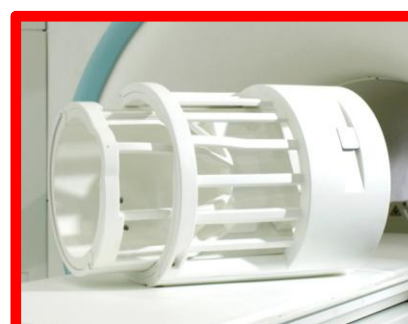
MR Scanner Components (simplified)

3 Types of magnetic fields:

1. Static field ($B_0 \parallel z$)
2. Dynamic gradients ($G_i = d$
3. **Radio-frequency** ($B_1 \perp z$)

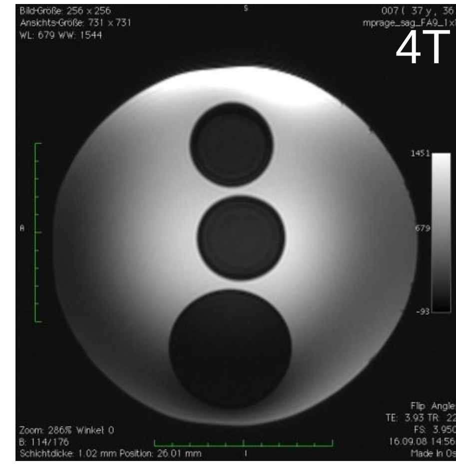


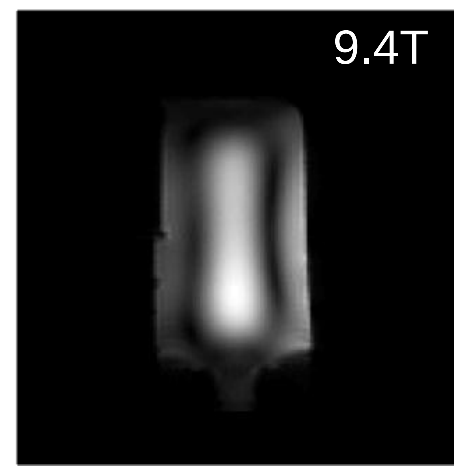




RF Considerations

- Larmor frequencies of ~45...150 MHz (Field strengths 1...3T) are common
- The sensitivity of the RF coil should be as homogeneous as possible in the Volume-Of-Interest (VOI)
- Siginificant problems at fields > 3T
- We will assume a perfectly homogeneous RF excitation





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Preliminaries

- Classical picture
- Ensemble of spins (sample)
- Polarized, i.e. sample placed in the magnetic field
- Equilibrium magnetisation aligned along the z direction

The Bloch equation without relaxation

Bloch equation

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_z - M_0}{T_1}\hat{\mathbf{z}} - \frac{M_x \hat{\mathbf{x}} - M_y \hat{\mathbf{y}}}{T_2}$$

Simplification

- *No relaxation*, consider processes much shorter than the relaxation times

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B}$$

Time varying RF field

- A linearly polarised amplitude modulated radiofrequency (RF) field transmitted at frequency ω

$$B_1(t) = \hat{B}_1(t) \cdot \cos(\omega t)$$

- This decomposes into two circularly polarised RF fields
- One rotating clockwise, one rotating counter clockwise

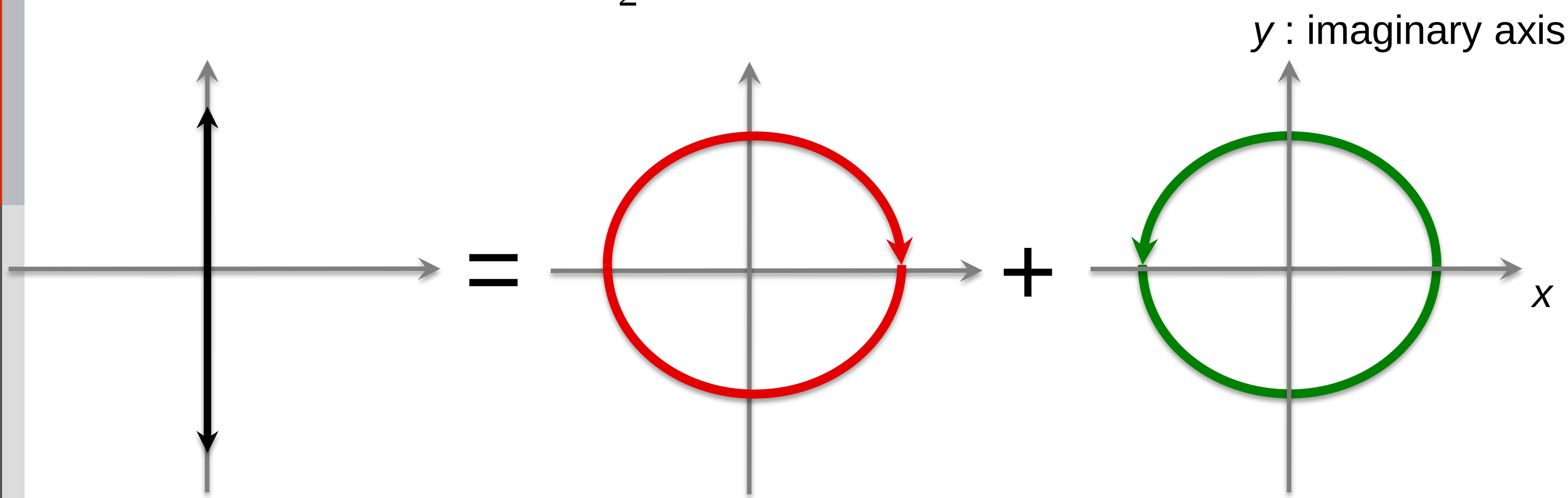
$$B_1(t) = \hat{B}_1(t) \cdot \cos(\omega t) = \frac{\hat{B}_1(t)}{2}\left(\underline{e^{-i\omega t}} + \underline{e^{+i\omega t}}\right)$$

- Only the left handed component affects the spins!

Time varying RF field

- Only the left handed component affects the spins!
- Same rotational direction as spin system ➔ Resonance, if ω=ω_0!

$$B_1(t) = \hat{B}_1(t) \cdot \cos(\omega t) = \frac{\hat{B}_1(t)}{2}\left(e^{-i\omega t} + e^{+i\omega t}\right)$$



Time varying RF field

- RF field in complex notation $\mathrm{B}_1(\boldsymbol{t}) = \hat{\boldsymbol{B}}_1 \cdot \boldsymbol{e}^{-i\omega t}$

- Vector notation $\boldsymbol{B}_{\boldsymbol{1}}(\boldsymbol{t}) = \hat{\boldsymbol{B}}_1(\boldsymbol{t}) \cdot (\cos(\boldsymbol{\omega t})\hat{\boldsymbol{x}} + \sin(\boldsymbol{\omega t})\hat{\boldsymbol{y}})$

- The Bloch equation without relaxation in matrix formulation for such an RF field

$$\frac{d\boldsymbol{M}}{dt} = \begin{pmatrix} 0 & \gamma \boldsymbol{B}_0 & \gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t})\sin\,\boldsymbol{\omega t} \\ -\gamma \boldsymbol{B}_0 & 0 & \gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t})\cos\,\boldsymbol{\omega t} \\ -\gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t})\sin\,\boldsymbol{\omega t} & \gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t})\cos\,\boldsymbol{\omega t} & 0 \end{pmatrix} \boldsymbol{M}$$

The rotating frame

- To simplify the above equations we move to the **rotating frame**

$$M \rightarrow M_{rot} \qquad B \rightarrow B_{rot}$$

$$x \rightarrow x_{rot} = x \cdot \cos(\omega t)$$
$$y \rightarrow y_{rot} = y \cdot \sin(\omega t)$$
$$z \rightarrow z_{rot} = z$$

- Bloch equation in the rotating frame

$$\frac{dM_{rot}}{dt} = M_{rot} \times \gamma B_{eff}$$

The Effective Field

- A time varying amplitude modulated RF field (complex notation) transforms according as (if the coordinate system rotates with identical frequency)

$$\boldsymbol{B}_1(\boldsymbol{t}) = \hat{\boldsymbol{B}}(\boldsymbol{t})\boldsymbol{e}^{-\boldsymbol{i\omega t}} \rightarrow \boldsymbol{B}_{1,\boldsymbol{rot}}(\boldsymbol{t}) = \hat{\boldsymbol{B}}(\boldsymbol{t})$$

- The B field acting in the rotating frame is given as (including *off-resonances*)

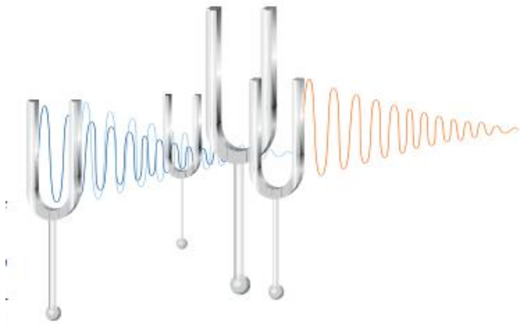
$$\boldsymbol{B}_{\boldsymbol{eff}} = \boldsymbol{B}_{1,\boldsymbol{rot}} + \underbrace{\left(\boldsymbol{B}_0 - \frac{\boldsymbol{\omega}_{\boldsymbol{rot}}}{\boldsymbol{\gamma}}\right)\hat{\boldsymbol{z}}}_{\text{Off resonances}}$$

Off resonances

On-Resonant Excitation

- In the case of *on-resonant* excitation

$$\boldsymbol{B}_{eff} = \boldsymbol{B}_{rot}$$



- We obtain

$$\frac{d\boldsymbol{M}_{rot}}{dt} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t}) \\ 0 & -\gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t}) & 0 \end{pmatrix} \boldsymbol{M}_{rot}$$

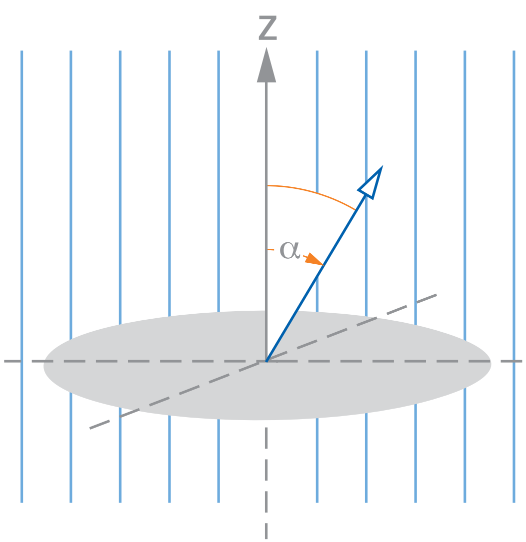
The Flip Angle

- Soution of is a rotation along the axis of the RF pulse (x in this case)

$$\boldsymbol{M}_{rot}(t) = \boldsymbol{R}_x(\alpha)\boldsymbol{M}_{rot}(0)$$

- The **flip angle** is given as

$$\alpha = \gamma \int_{-\tau/2}^{+\tau/2} \boldsymbol{B}_1(t)\,dt$$



The Rectangular Pulse

- RF pulses with constant amplitude, phase and finite duration are called **Rectangular Pulses**
- In this case the flip angle is given as

$$\alpha = \gamma B_1 \tau$$

QUIZ!

We assume a spin ensemble initially at equilibrium – *which flip angle maximises the amount of transverse magnetisation*?

1) 45°
2) 90°
3) 270°

QUIZ!

You generate a rectangular RF pulse that gives rise to a flip angle of 90°.

How can you double the flip angle?

1) Double the duration
2) Double the amplitude
3) Double amplitude and duration simultaneously
4) None of the above

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Off-Resonance

- Till now we considered the case of *on-resonant* excitation
- The carrier frequency of the RF field was tuned to the larmor frequency
- System imperfections (static field inhomogeneities) and sample properties (chemical shift, susceptibility) give rise to spatially varying Larmor frequencies
- Usually off-resonances are modelled as (small) additional contributions to B_0
- Off-resonance effects on excitation

Off-Resonance

- Recall equation – off-resonances are contributions to the z component of B_{eff}

$$\boldsymbol{B}_{eff} = \boldsymbol{B}_{rot} + \left(\boldsymbol{B}_0 - \frac{\boldsymbol{\omega}_{rot}}{\gamma} \right) \hat{\boldsymbol{z}}$$

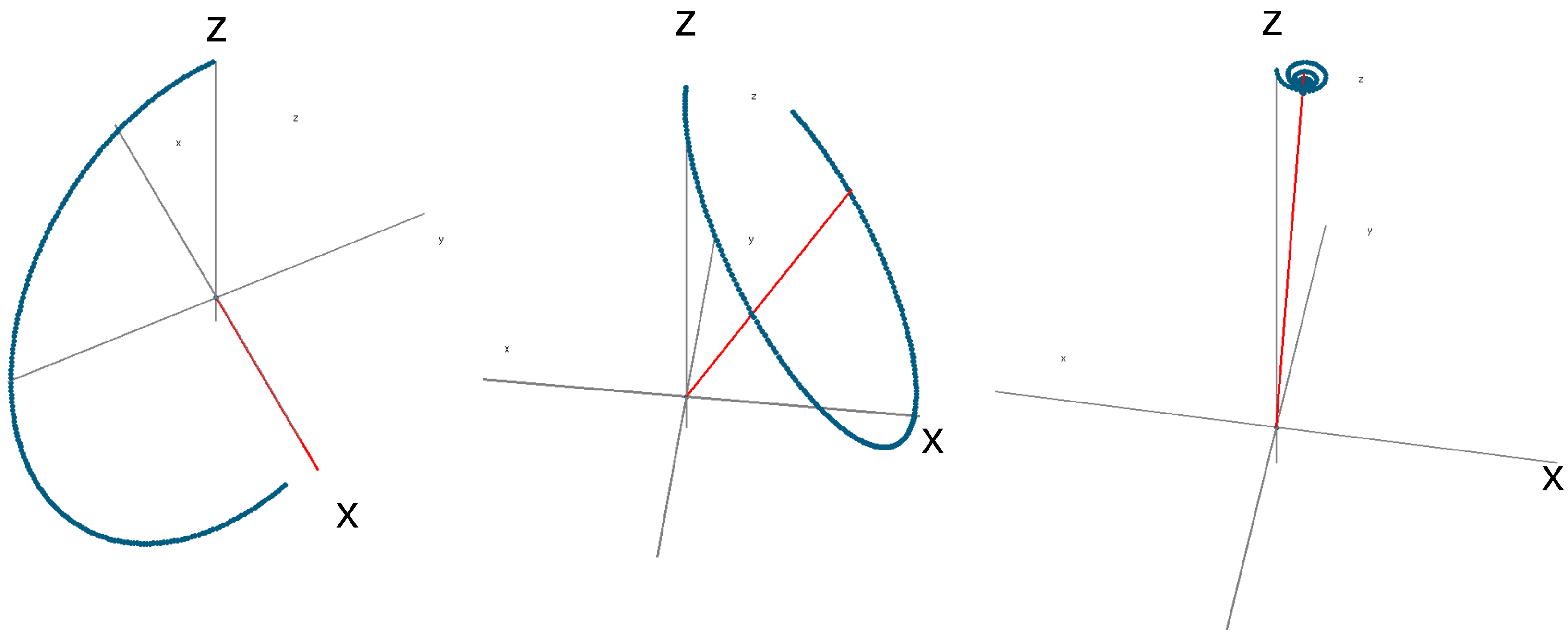
$$\gamma \boldsymbol{B}_0 - \boldsymbol{\omega}_{rot} \equiv \Delta \boldsymbol{\omega}$$

- Now consider the *off-resonance* term – **no *general* solution**

$$\frac{d\boldsymbol{M}_{rot}}{dt} = \begin{pmatrix} 0 & \Delta\boldsymbol{\omega} & 0 \\ -\Delta\boldsymbol{\omega} & 0 & \gamma \hat{\mathrm{B}}_1(\boldsymbol{t}) \\ 0 & -\gamma \hat{\mathrm{B}}_1(\boldsymbol{t}) & 0 \end{pmatrix} \boldsymbol{M}_{rot}$$

Off-Resonance – examples

- **The excitation is a precession around the effective field**

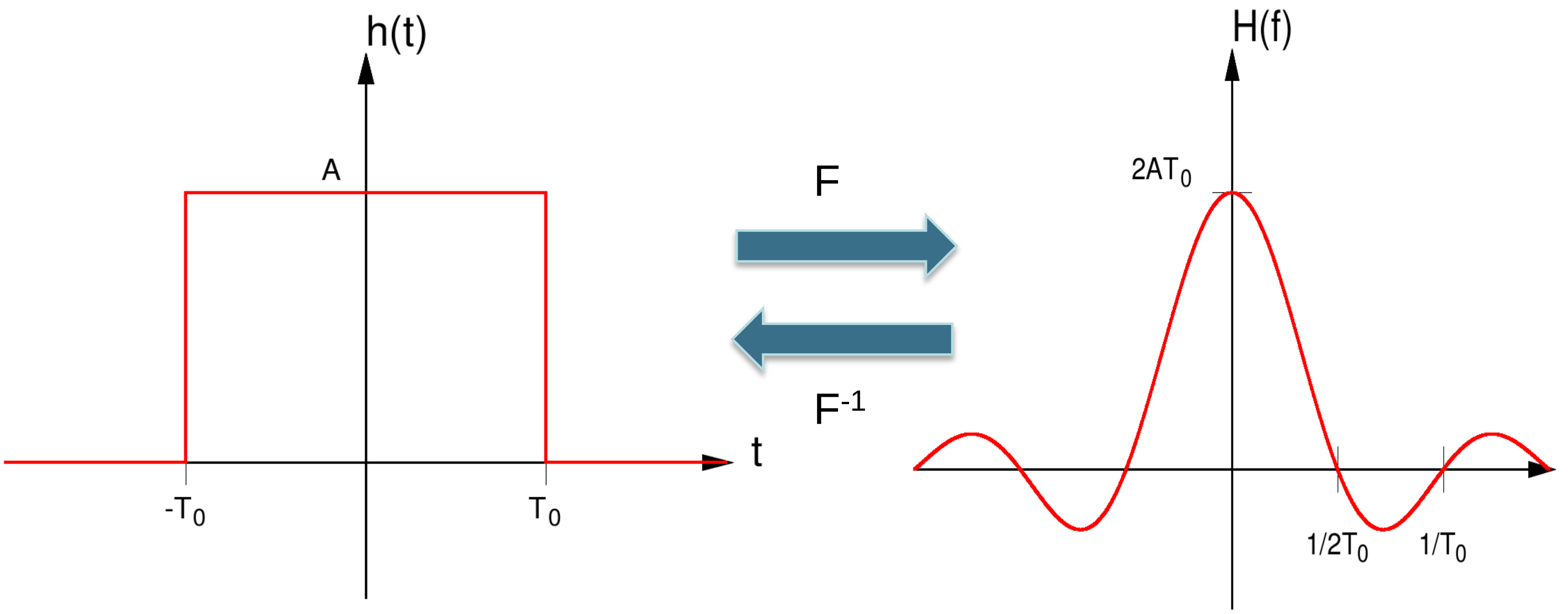


No off-resonance

Medium off-resonance

Large off-resonance

Off-Resonance – Frequency Response of a RECT Pulse



The Effects of Off-Resonance – Summary

- Off-resonant spins are excited *less effective,* i.e. the flip angle is lower than without off-resonances
- Off-resonant excitation usually gives rise to a non-zero phase of the transverse magnetisation
- In the case of too large off-resonances no excitation takes place
- Controlled off-resonances (gradients) are valuable to select parts of the sample to be excited

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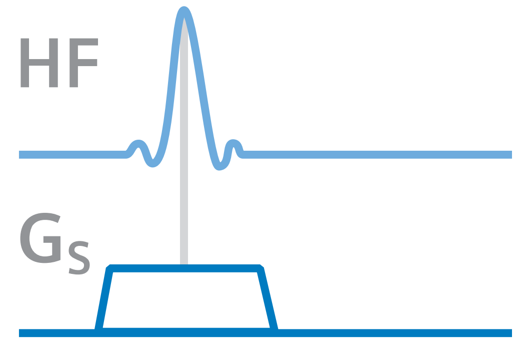
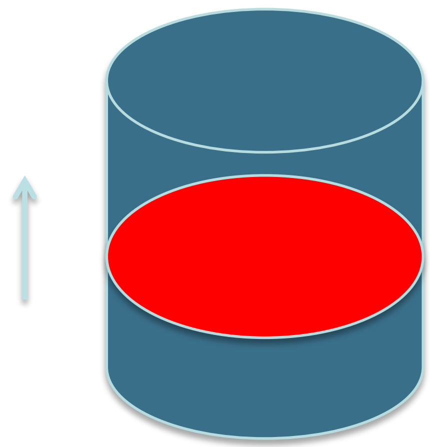
Silce Selection - Introduction

- Shortens the acquisition time
- Simultaneous application of gradients and RF pulses
- Slices can be created in arbitrary orientations (*double oblique*) by simultaneous application of several gradients





Silce Selection - Gradients

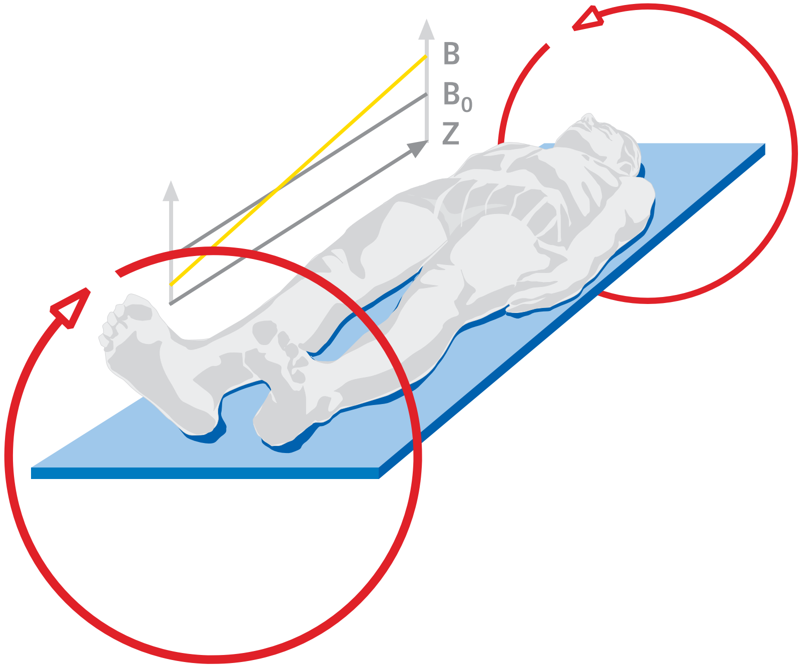
- *Gradient* in MRI usually denotes a linear field gradient of the z-component of the B field (e.g. for x direction)

$$\frac{\partial B}{\partial x} \equiv \frac{\partial B_z}{\partial x} \equiv G_x$$

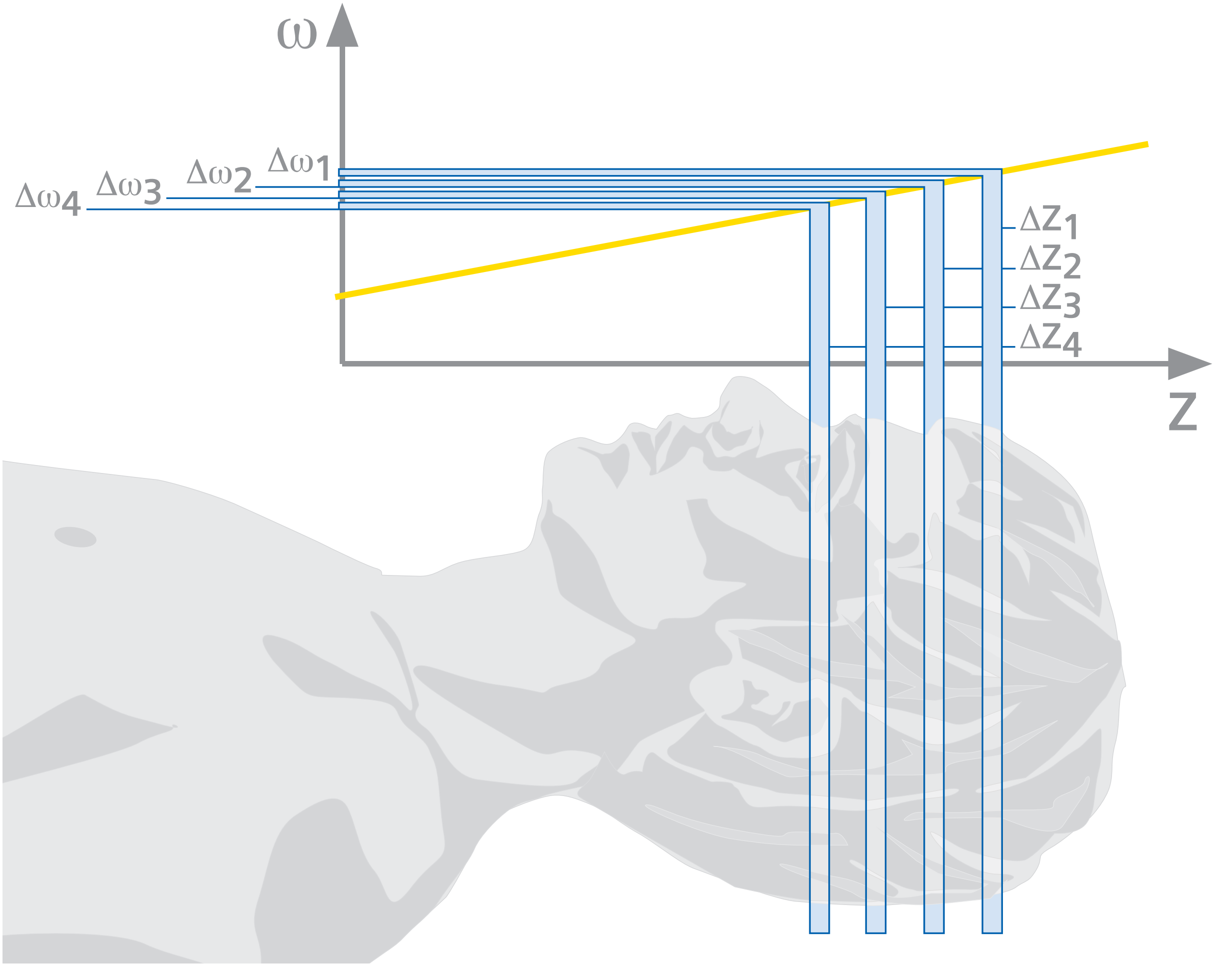
- Order of magnitude is usually ‚*milli Tesla per metre*‘ (mT/m)
- Together with the RF field (only transverse components) the time dependent magnetic field becomes

$$\boldsymbol{B}(t) = \boldsymbol{B}_1(t) + (B_0 + \boldsymbol{G}(t) \cdot \boldsymbol{r})\hat{\boldsymbol{z}}$$

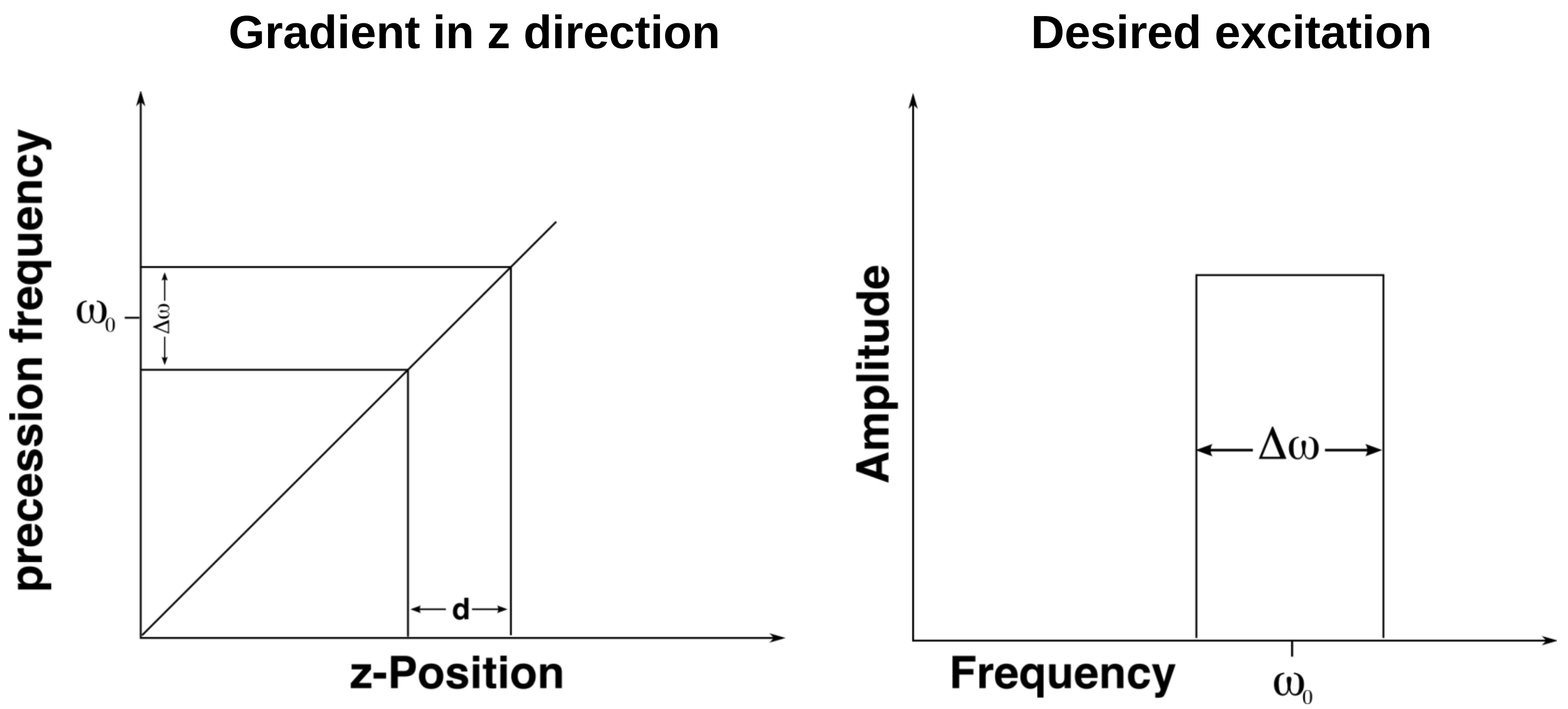
Slice Selection - Motivation



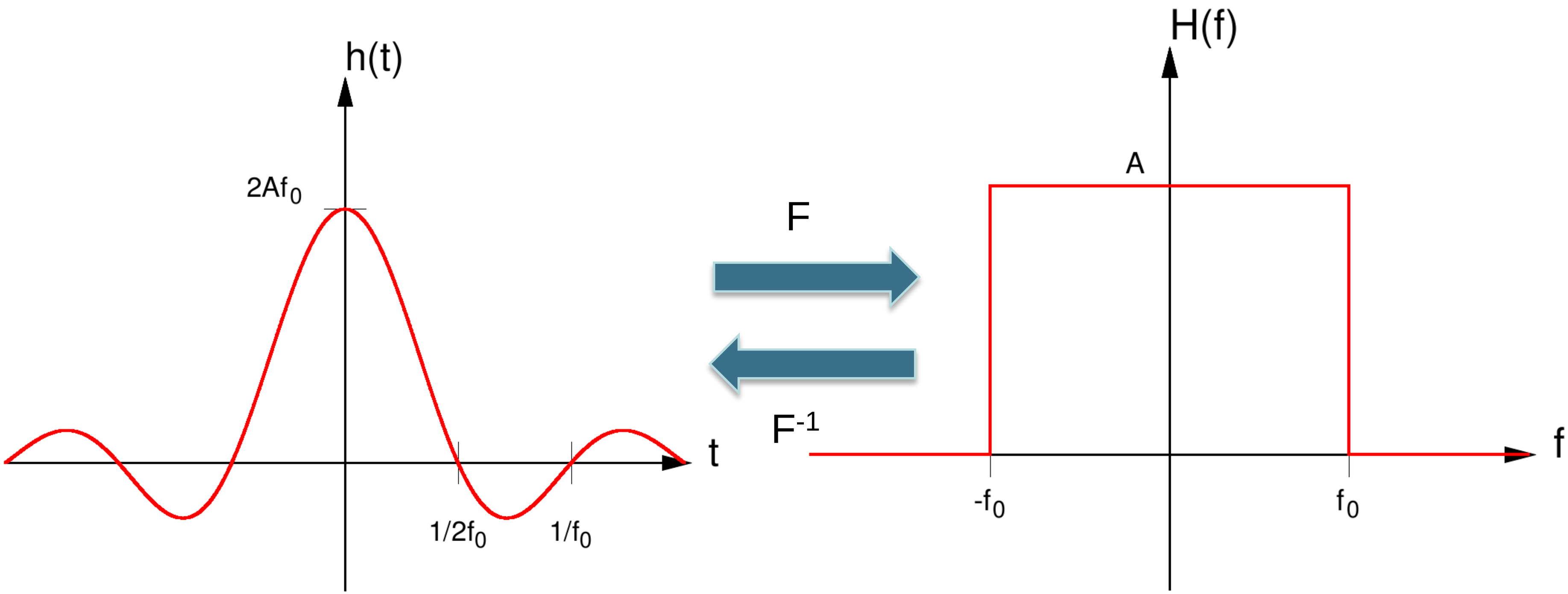
Slice Selection - Motivation



Slice Selection – Pictorial Description



How to Excite the Desired (Rectangular) Profile?



RF pulse

Frequency response

Slice Selection – Slice Thickness

- The RF bandwidth Δf determines the *amount of frequencies* contained in a pulse
- The slice thickness is determined by the frequency band of the excitation pulse (as a function of gradient amplitude)

$$\Delta z = \frac{2\pi\Delta f}{\gamma G_z}$$

- In case of RF pulses without a rectangular freq. response (most pulses) the *Full-Width-Half-Maximum (FWHM)* is usually defined as bandwidth

JÜLICH FORSCHUNGSZENTRUM

Slice Selection – Bloch Equation With Gradients

- Deliberately introduce off-resonances by applying a z gradient (no other sources of off-resonance)

$$\boldsymbol{B}_{eff}(\boldsymbol{z},\boldsymbol{t}) = \boldsymbol{B}_1(\boldsymbol{t}) + \boldsymbol{G}_z \boldsymbol{z}\,\hat{\boldsymbol{z}}$$

- Matrix formulation – again, no easy solution

$$\frac{d\boldsymbol{M}_{rot}(\boldsymbol{z})}{dt} = \begin{pmatrix} 0 & \gamma \boldsymbol{G}_z \boldsymbol{z} & 0 \\ -\gamma \boldsymbol{G}_z \boldsymbol{z} & 0 & \gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t}) \\ 0 & -\gamma\,\hat{\mathrm{B}}_1(\boldsymbol{t}) & 0 \end{pmatrix} \boldsymbol{M}_{rot}(\boldsymbol{z})$$

Slice Selection – Small Tip Angle Approximation

- Simplify for analytical treatment
  - *Small Tip Angle ($\sin(\alpha)=\alpha$)*
  - *Constant longitudinal magnetisation ($M_z(t) = M_0$)*

$$\frac{d\mathbf{M}_{rot}(z)}{dt} = \begin{pmatrix} 0 & \gamma G_z z & 0 \\ -\gamma G_z z & 0 & \gamma \hat{B}_1(t) \\ 0 & 0 & 0 \end{pmatrix} \mathbf{M}_{rot}(z)$$

Longitudinal and transverse magnetisation are decoupled!

Slice Selection – Small Tip Angle Approximation

- Rewrite in complex notation – first order nonlinear differential equation

$$\frac{dM}{dt} = i\gamma G_z zM + i\gamma B_1(t)M_0$$

- Solved by

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

$$M_{xy}(t,z) = iM_0 e^{-i\omega(z)t} \int_0^t e^{i\omega(z)t'} \omega_1(t')dt'$$

Slice Selection – Small Tip Angle Approximation

- After a RF pulse of duration τ

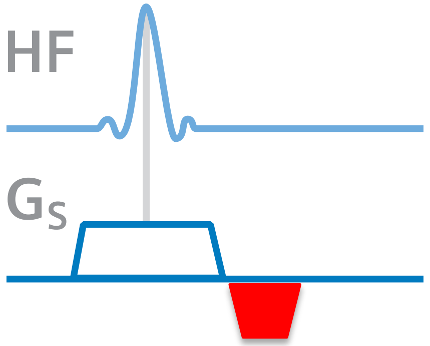
$$M_{xy}(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \underline{\int_{-\tau/2}^{+\tau/2} e^{i\omega(z)t'} \omega_1\left(t' + \frac{\tau}{2}\right) dt'}$$

- The second term is identified as the Fourier transform of the applied RF pulse shape

$$M_{xy}(\tau, z) = ie^{-i\omega(z)\tau/2} M_0 F\left\{\omega_1\left(t + \frac{\tau}{2}\right)\right\}$$

Slice Selection – Rephasing Gradient

- After the pulse of duration tau



$$M_{xy}(\tau, z) = i e^{-i\omega(z)\tau/2} M_0 F\left(\omega_1 (t' + \frac{\tau}{2})\right)$$

- Dephasing term leads to signal cancellation
- Dephasing is linear in z and is, therefore, removed by the application of a gradient with opposite amplitude and half duration

Sice Selection – *Take Home Message*

In the *Small Tip Angle Approximation* the slice profile is given by the Fourier transform of the RF pulse shape

QUIZ!

Assume a SINC pulse with a fixed freqency response in the presence of a constant gradient in z direction.

How can you move the position of the excited slice?

1) Change the carrier frequency of the pulse
2) Change the gradient amplitude
3) Change the RF amplitude
4) None of the above

QUIZ!

Same situation – *how can you change the thickness of the slice?*

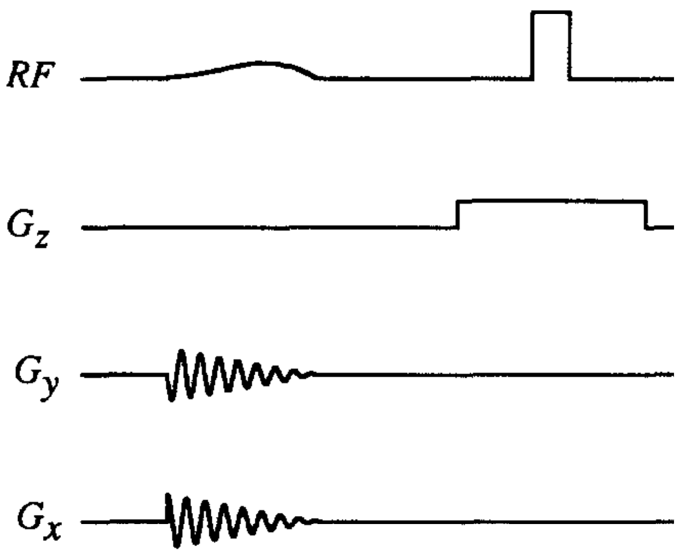
1) Change the carrier frequency of the pulse
2) Change the gradient amplitude
3) Change the RF amplitude
4) None of the above

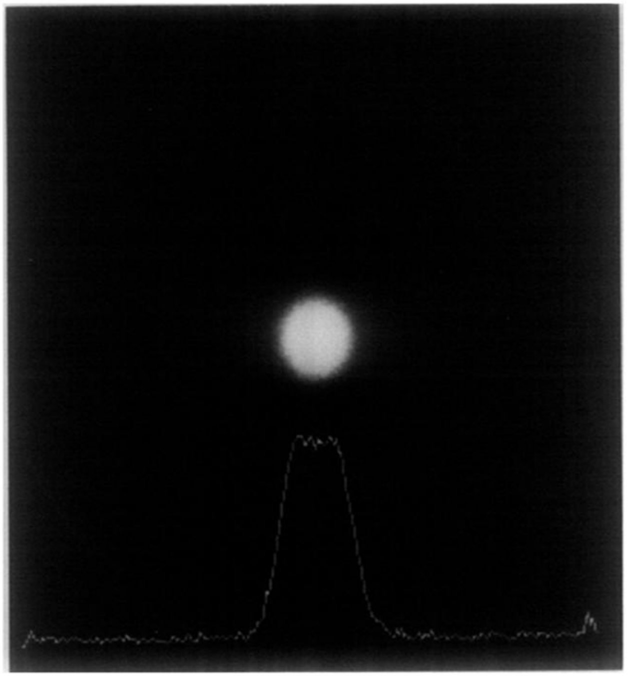
Generalized Selective Excitation

- Selective excitation is not limited to slices (1D)
- Can be extended to 2D or even higher dimensionality (3D, spatially-spectral-selective)
- Design of selective pulses is sophisticated and area of active research

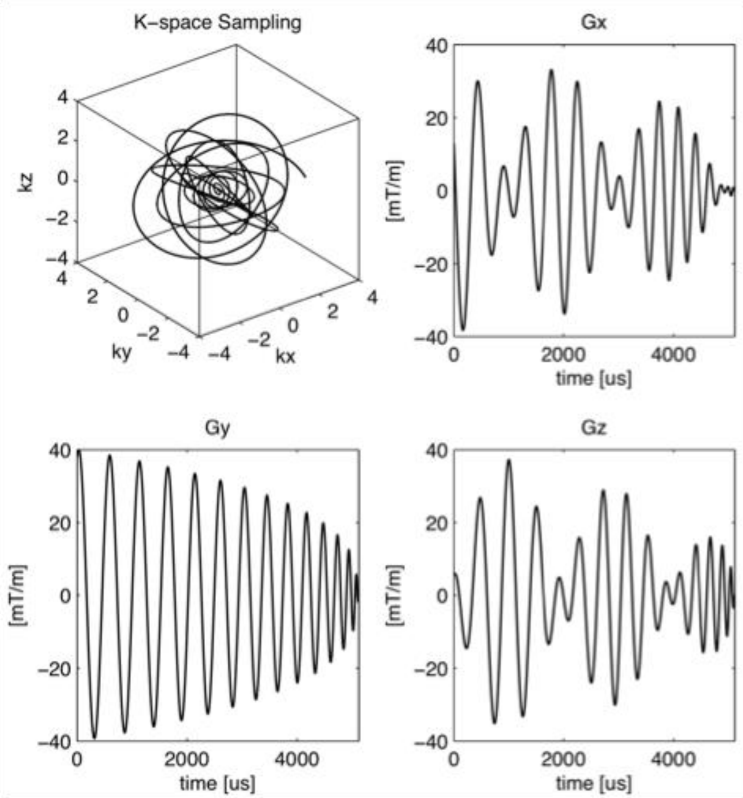
Example: Pencil Beam Excitation

Pauly et al.; Journal of Magnetic Resonance 81,43-56 (1989)





Example: 3D Selective Excitation



Vahedipour et al., FZ Juelich, 2010

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Safety Considerations of RF Excitations

- Specified in IEC 60601-2-33 norm
- 4 W/kg averaged over the whole body for any 15-minute period
- 3 W/kg averaged over the head for any 10 minute period
- 8 W/kg in any gram of tissue in the extremeties for any 5 minute period.
- Corresponding to a maximum tissue heating of 1°C
- Not applicabale for UHF (> 4T) due to SAR localisations, to date (10/2010) no norm exists

SAR Calculations

- SAR is proportional to the square of the RF amplitude
- SAR is proportional to the square of the larmor frequency (and, thus, the static field)

$$SAR \propto B_0^2 \hat{B}_1^2$$

- **Implications**
  - At higher field strengths it is necessary to reduce B_1
  - To maintain the specified flip angle one has to prolongue the RF pulses

Summary

- Excitation is performed by the application of RF pulses transmitted at larmor frequency
- The flip angle is proportional to the amplitude of B_1 and pulse duration for non-selective excitation
- Selective excitation is performed to limit the volume that has to be imaged – to shorten acquisition times
- The profile of the excited slice is given by the Fourier transform of the RF pulse
- SAR limits impose a minimum time between RF pulses