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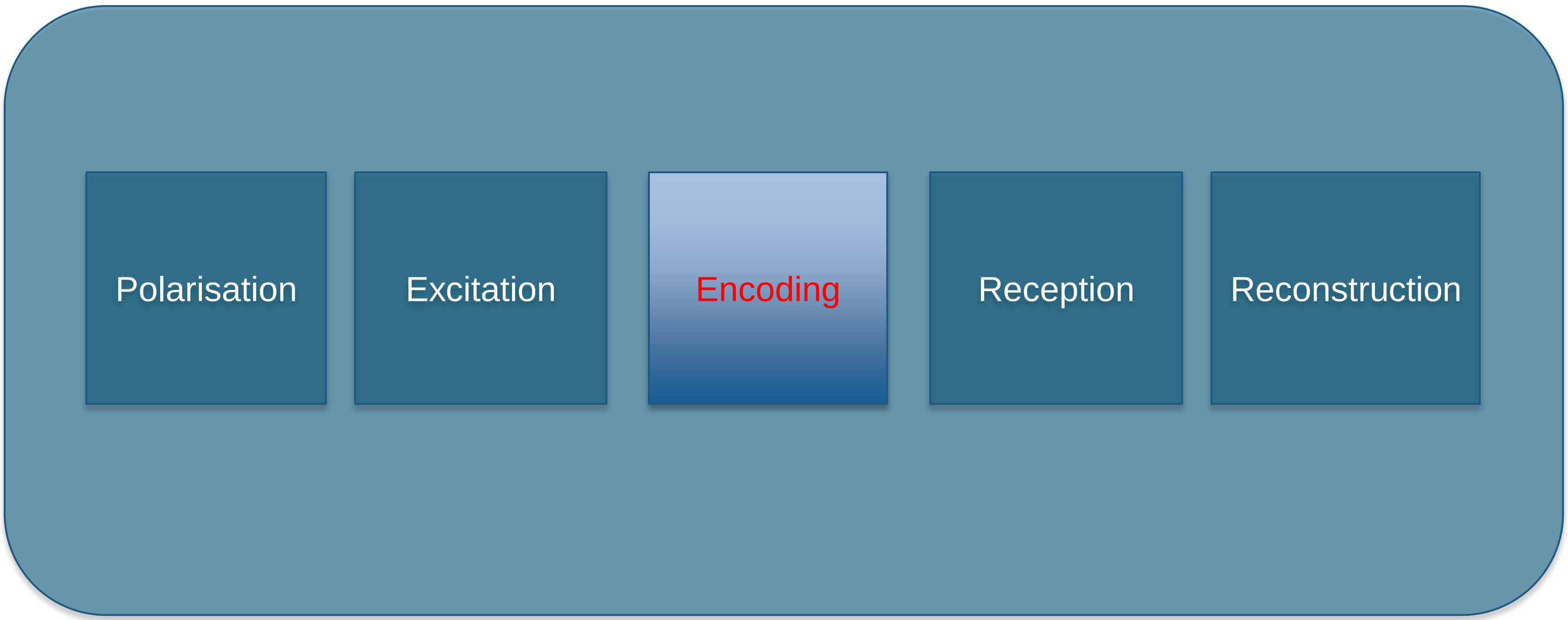
Imaging Principles 1

The Whole Picture



Steps of an MRI experiment

Selected Solutions of the Bloch Equations

Rewriting the Bloch Equations in matrix formulation:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_z - M_0}{T_1}\mathbf{e}_z - \frac{M_x \mathbf{e}_x + M_y \mathbf{e}_y}{T_2}$$

$$\Rightarrow \quad \frac{d}{dt}\begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} = \begin{pmatrix} -1/T_2 & \gamma B_z & -\gamma B_y \\ -\gamma B_z & -1/T_2 & \gamma B_x \\ \gamma B_y & -\gamma B_x & -1/T_1 \end{pmatrix} \cdot \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ M_0/T_1 \end{pmatrix}$$

- No analytic solution for general B field, $\mathbf{B}(t)=B_x(t)\mathbf{e}_x+B_y(t)\mathbf{e}_y+B_z(t)\mathbf{e}_z$
- If $\mathbf{B} || \mathbf{e}_z$ ($B_x=B_y=0$) the Bloch equations decouple: simple solutions
- These solutions are important for the encoding step (after excitation!)

Selected Solutions of the Bloch Equations

static field $\mathbf{B} = B_o\mathbf{e_z}$ and $\mathbf{M}(t=0)=M_0\mathbf{e_x}$ (after 90° pulse)

$$\frac{dM}{dt} = \begin{pmatrix} -\frac{1}{T_2} & \gamma B_0 & 0 \\ -\gamma B_0 & -\frac{1}{T_2} & 0 \\ 0 & 0 & -\frac{1}{T_1} \end{pmatrix} M + \begin{pmatrix} 0 \\ 0 \\ \frac{M_0}{T_1} \end{pmatrix}$$

$$M(t) = \begin{pmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T1} \end{pmatrix} R_z(\omega_0 t) M_0 + \begin{pmatrix} 0 \\ 0 \\ M_0\left(1 - e^{-t/T_1}\right) \end{pmatrix}$$

Selected Solutions of the Bloch Equations

static field $\mathbf{B} = B_o \mathbf{e_z}$ and $\mathbf{M}(t=0)=M_0\mathbf{e_x}$ (after 90° pulse)

Complex form:

$$\frac{dM_\perp}{dt} = \frac{dM_x}{dt} + i\frac{dM_y}{dt} = -\left(\frac{1}{T_2} + i\omega_0\right) M_\perp$$

$$M_\perp(t) = M_0 e^{-t/T_2} e^{-i\omega_0 t}$$

Selected Solutions of the Bloch Equations

Time-varying gradient fields $B(t)\vec{e}_z = (\vec{G}(t)\cdot\vec{r})\vec{e}_z$

$$M_\perp(t) = M_0 \exp\left[-\frac{t}{T_2} - i\omega_0 t - i\gamma\int_0^t \vec{G}(\tau)\cdot\vec{r}\,d\tau\right]$$

with $\vec{k}(t) \equiv \gamma\int_0^t \vec{G}(\tau)d\tau$ unit: m^{-1} (spatial frequency)

follows $$M_\perp(t) = M_0 \mathrm{e}^{-t/T_2}\mathrm{e}^{-i\omega_0 t}\mathrm{e}^{-i\vec{k}(t)\cdot\vec{r}}$$

MRI Signal Equation

If a time-varying gradient field is applied, then the MR signal is the Fourier Transform (FT) of the proton density distribution $M_0(\boldsymbol{r})$ (ignoring relaxation for now)

$$S(t) = \int_{\text{object}} M_\perp(\vec{r},t)\,d^3r$$

$$= e^{-i\omega_0 t} \int_{\text{object}} M_0(\vec{r})\,e^{-i\vec{k}(t)\cdot\vec{r}}\,d^3r$$

$$= \mathrm{FT}[M_0(\vec{r})]_{\vec{r}\to\vec{k}} \equiv S(\vec{k})$$

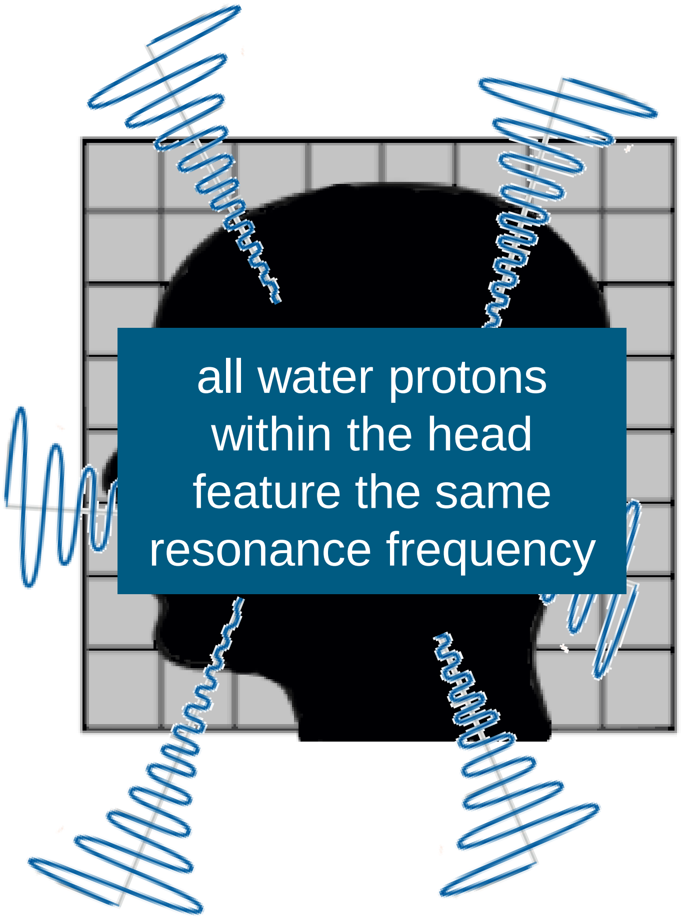
where $\vec{k}(t) \equiv \gamma \int_0^t \vec{G}(\tau)\,d\tau$ unit: m^{-1} (spatial frequency)

denotes the k-space trajectory

Imaging ?

During a (simple) MR experiment (e.g. FID, SE) an MR signal is received, which is the sum of all nuclear magnetic resonances within the entire sample.

However, since we do not have a spatial allocation, we cannot distinguish between the signals of different tissue structures.



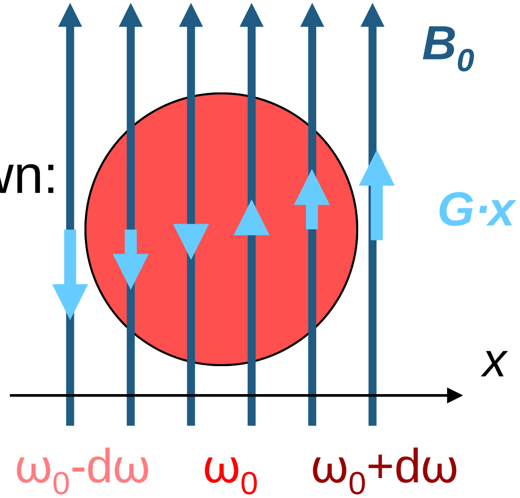
Source: Siemens, Magnete, Spins und Resonanzen

Imaging: Spatial Encoding in Time

Remember: The basic idea of MRI

Make the precessional frequency a function of space!

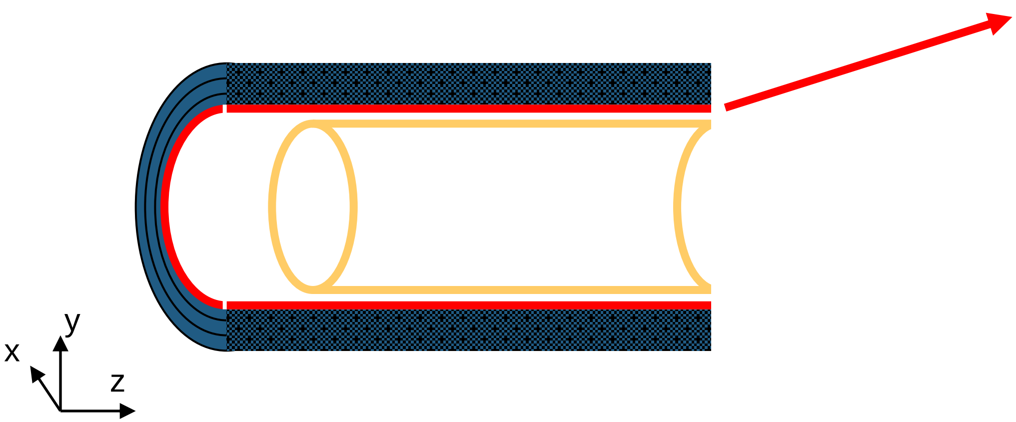
- The “spectrum” then reflects spatial distribution.
- Linear field gradients of the B-field in z-direction, e.g. $G_x = dB_z/dx$
- One trick to get spatial information is already known:
  - ➔ Slice selective excitation
- But there are 2 dimensions left…

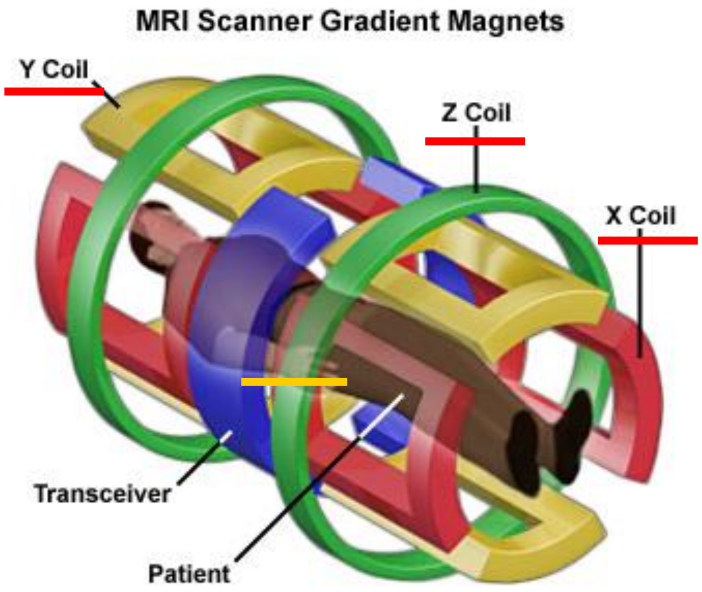


MR Scanner Components (simplified)

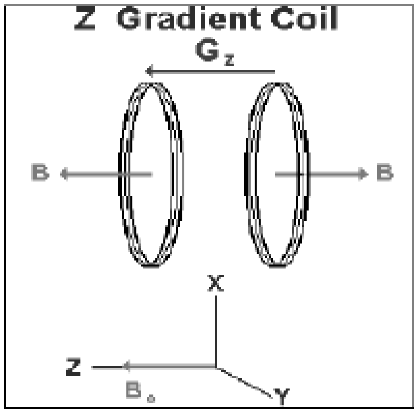
3 Types of magnetic fields:

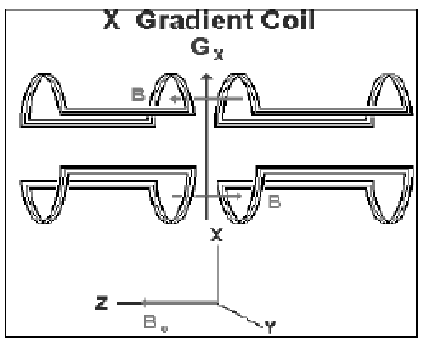
1. Static field ($B_0 \parallel z$)
2. **Dynamic "gradients"** ($G_i = dB_z/dr_i$)
3. Radio-frequency ($B_1 \perp z$)

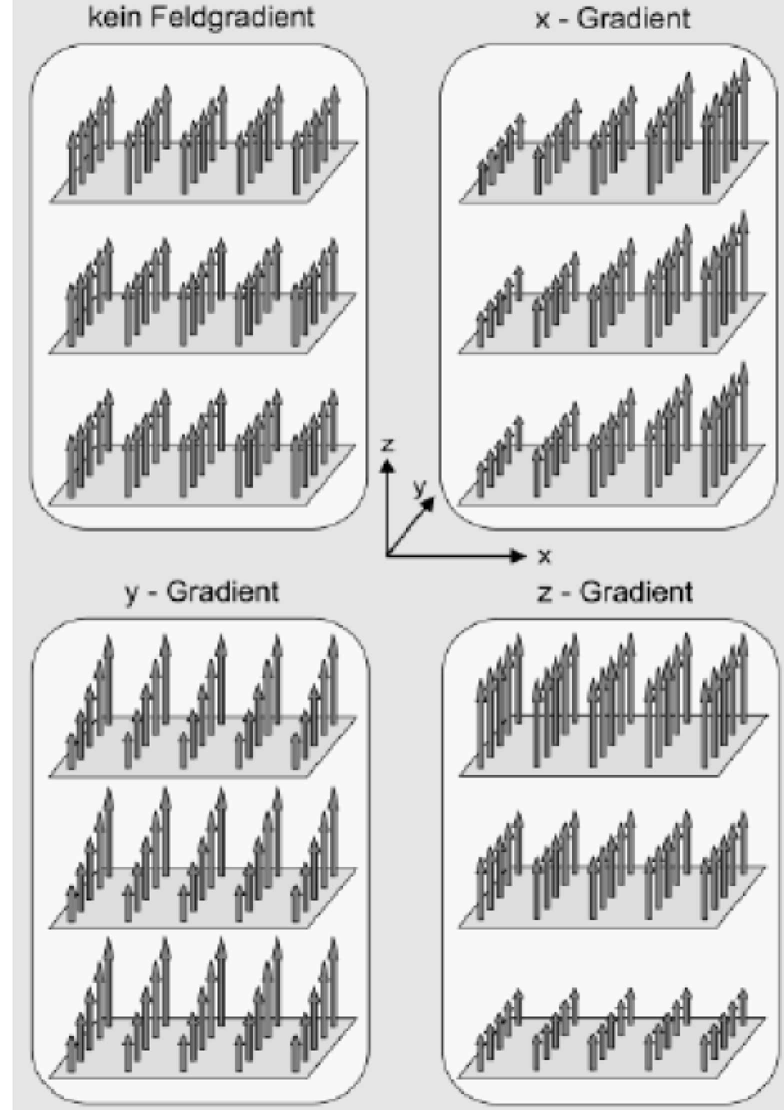




Gradient Coils: linear field variation







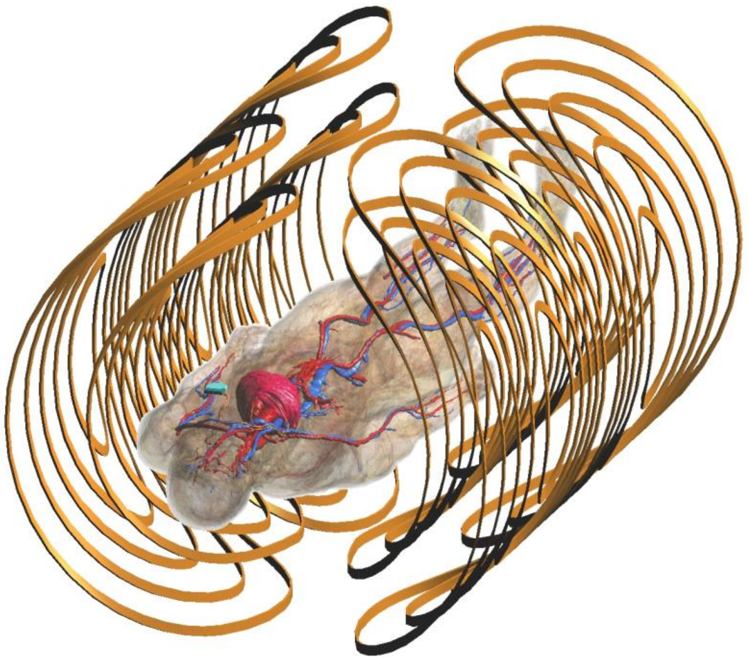
$$\Rightarrow \mathbf{B}(t) = (B_0 + \mathbf{G}(t)\cdot\mathbf{r})\mathbf{e}_z$$

Gradient Coils: some facts

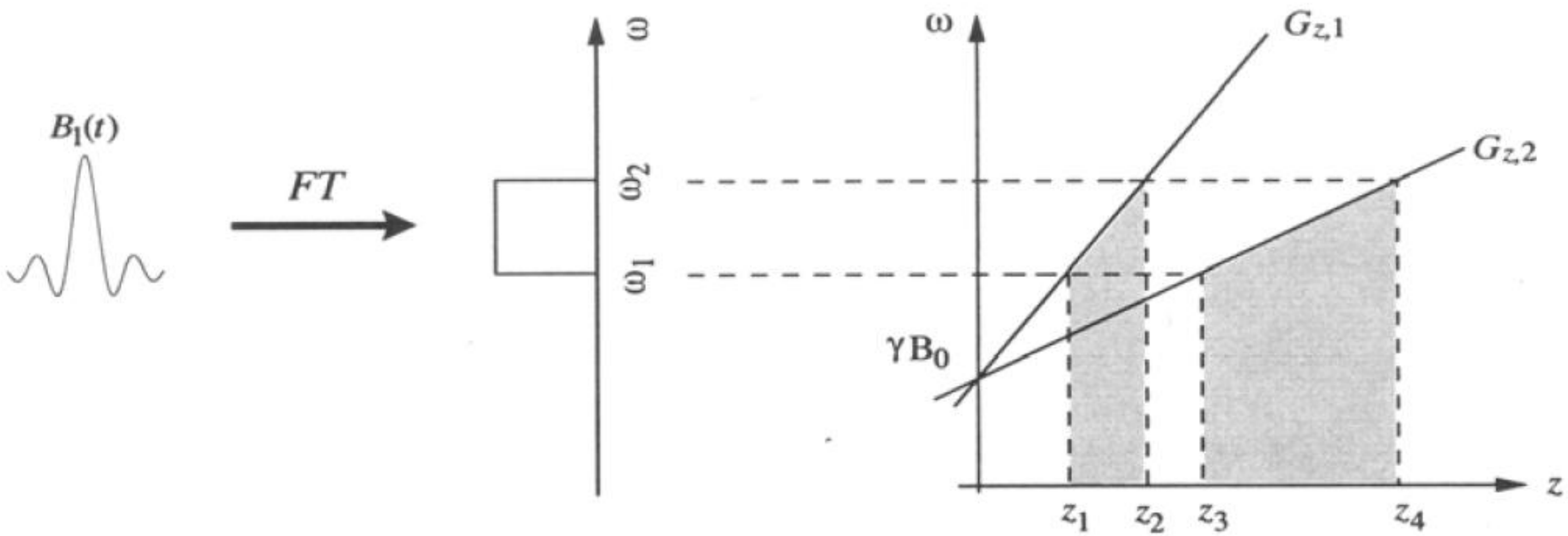
- typical range: 1-40 × 10^{-3} T/m

  e.g.: x=30 cm, B_0=1T, G=10 mT/m

  ➔ B = 0.9985 T … 1.0015 T
- rise time: 200-600 µs
- linearity: 40-60 cm
- power: e.g. 500 A at 2000 V in short time, power in MW range
- need liquid cooling
- “make the noise”



Remember:
Slice Selection

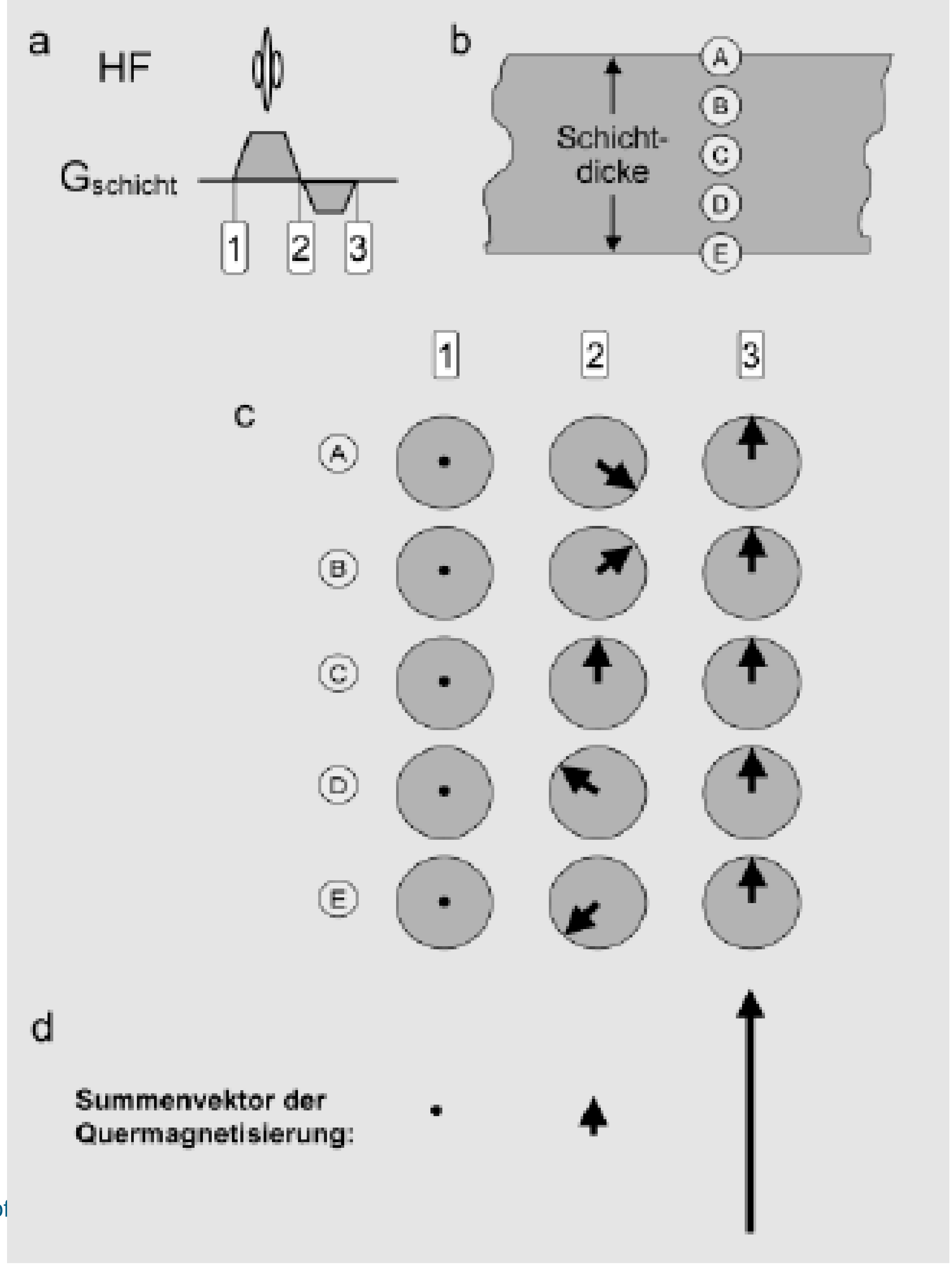


- Sinc-Pulse generates rectangular frequency distribution
- different gradient strength leads to slices with different thickness
- problem: Larmor frequency is different within slice thickness

Remember: Slice Selection

Slice Refocusing:

- phase dispersion factor is a linear function of position
- it is removed by the application of opposite *z*-gradient that produces a phase factor of $\exp(i\ \gamma\ G\ z\ \tau/2)$
- the gradient pulse is called “refocusing lobe” or “slice rewinder”



Frequency Encoding

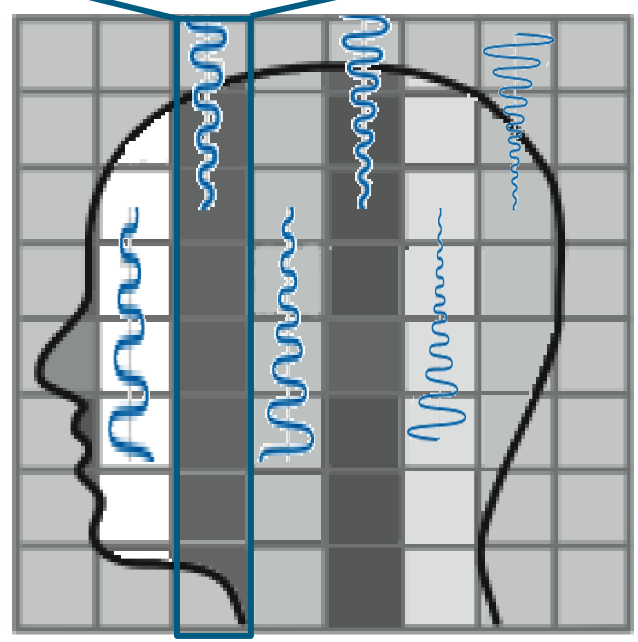
- Idea: Use the fact, that the resonance frequency depends on the field strength

- Can we spatially modify the resonance frequency?

- **<u>Frequency encoding</u>: apply gradient during data acquisition**

$$\Delta\omega = \gamma G_x x$$

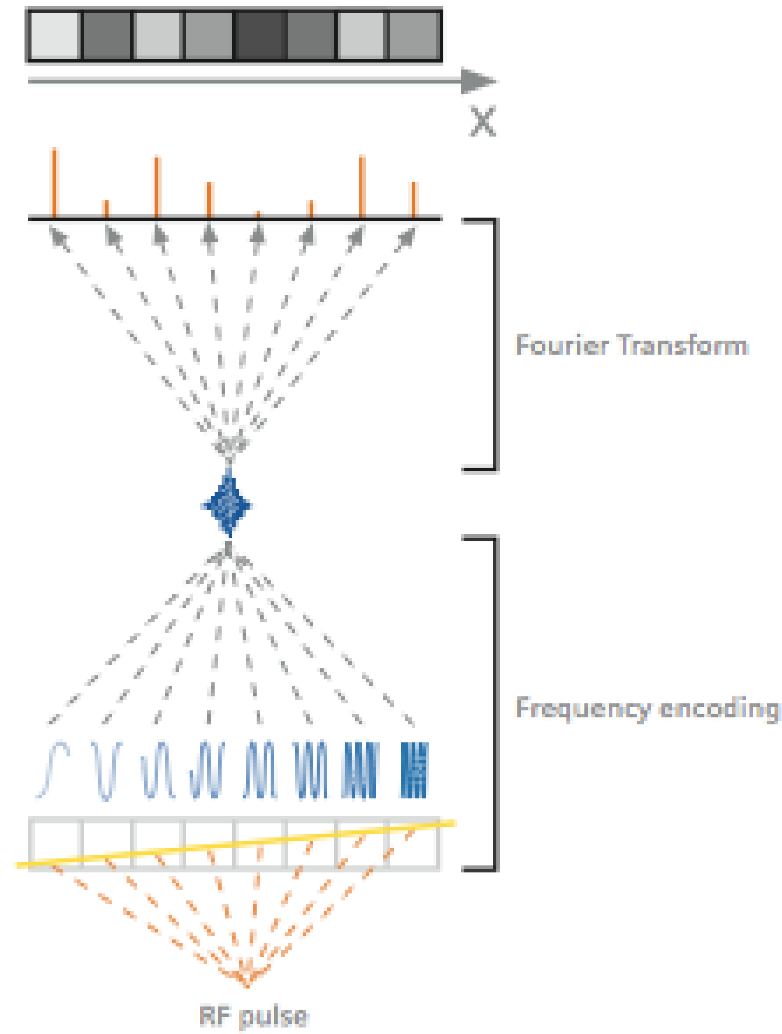
$$M_{\perp}^{n}(t) = M_0 e^{-\frac{t}{T_2}} e^{-i\left(\omega_0 + \Delta\omega^n\right)t}$$



Source: Siemens, Magnete, Spins und Resonanzen

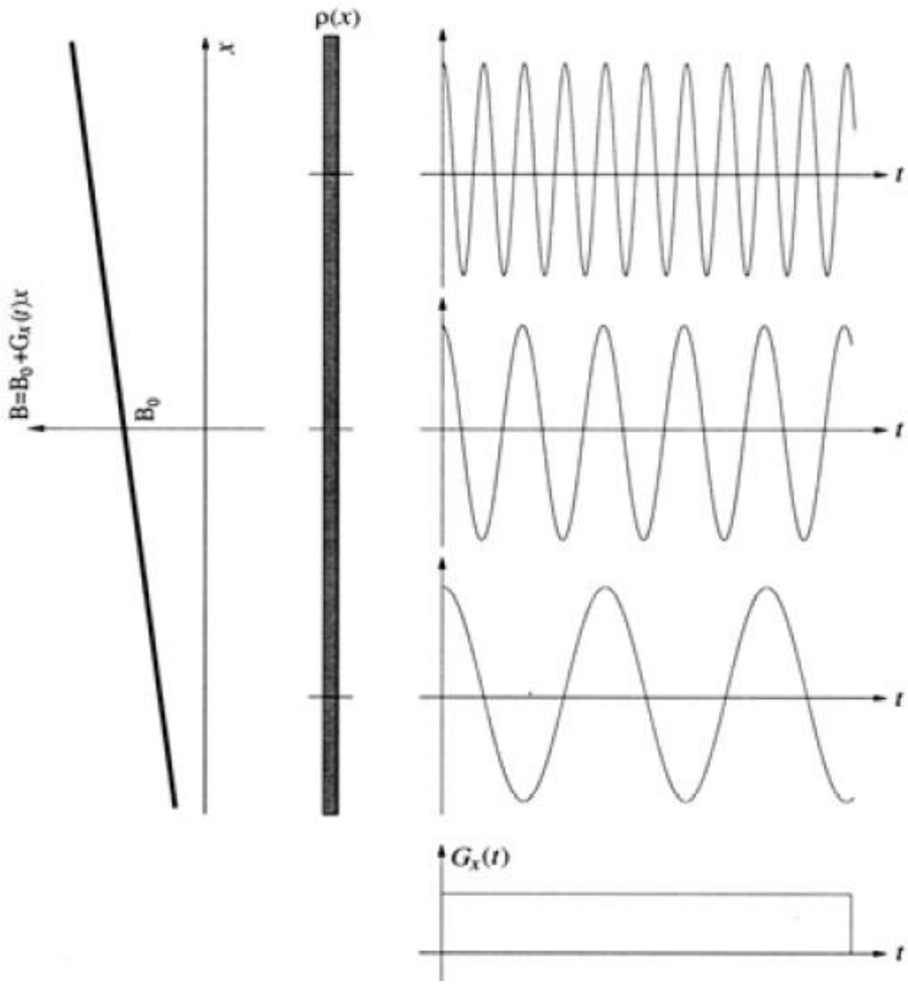
Frequency Encoding

- Employing the Fourier Transformation allows one to interpret frequency information as spatial location



Source: Siemens, Magnete, Spins und Resonanzen

Frequency Encoding



Larmor frequency is linearly dependent on spatial coordinate:

$$\omega(x) = \gamma(B_0 + G_x \cdot x)$$

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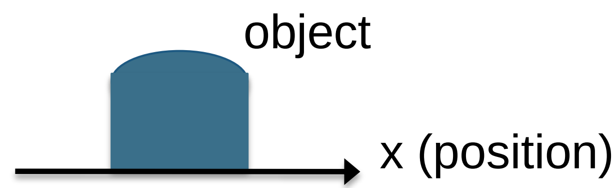
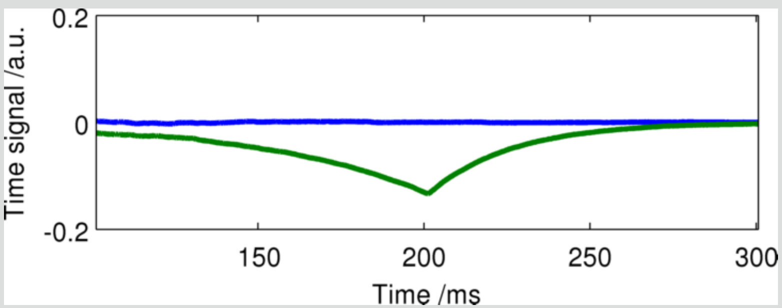
Frequency Encoding

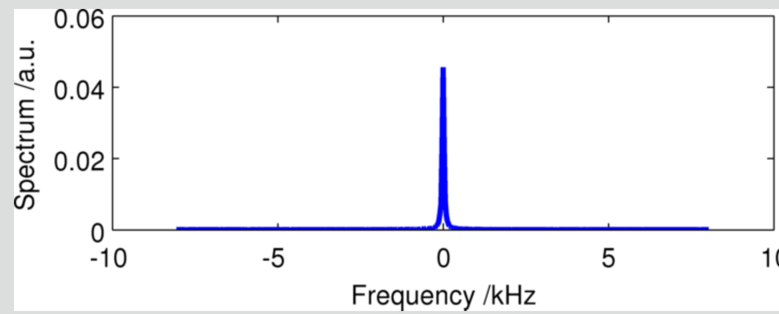


MR signal of homogeneous sample with uniform field B_0

Signal (demodulated)

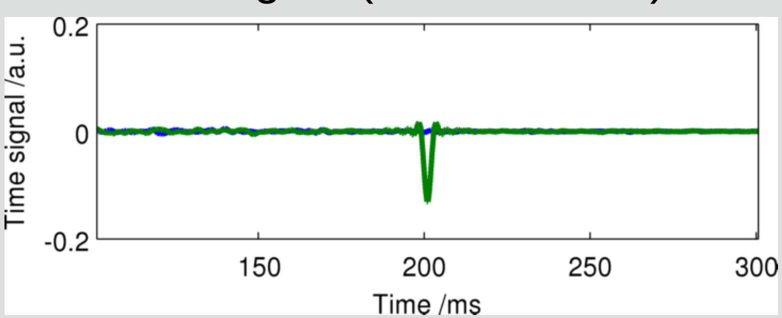


Spectrum

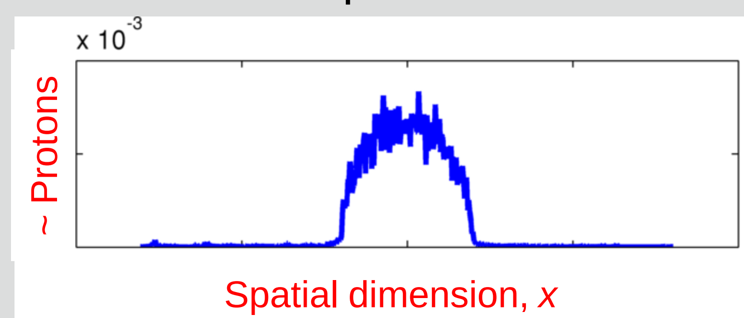


Field Gradient $G_x = dB_z/dx$ during signal acquisition:

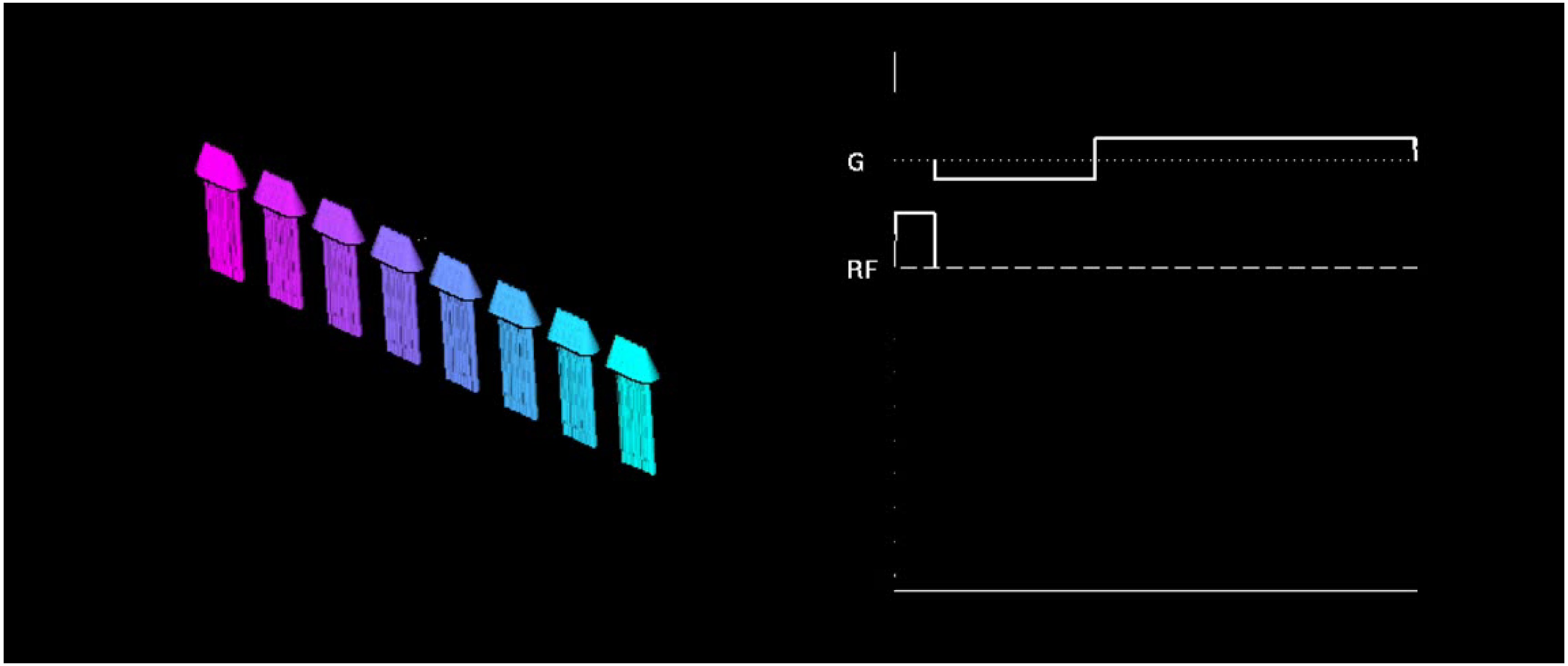
Signal (demodulated)



Spectrum



Frequency Encoding



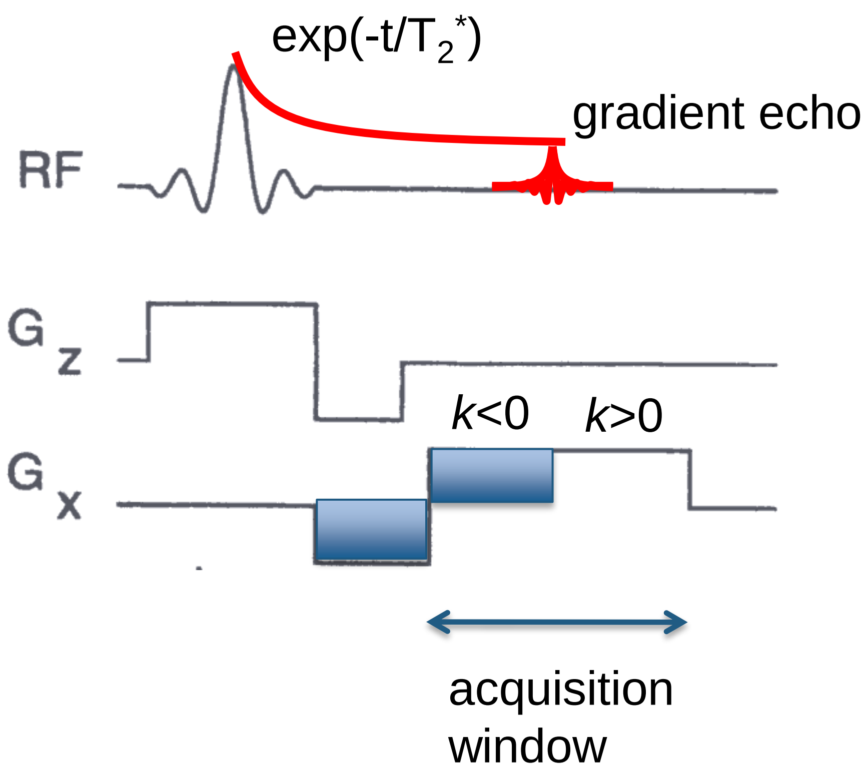
Source: Siemens, Magnete, Spins und Resonanzen

Frequency Encoding: Sampling

- MR signal S(t) is digitalized by using an "analog to digital converter" ADC and a discrete timing interval Δt in a total acquisition window t_{aq}
- For the Fourier-analysis of the MR signal there are in total $N = t_{aq}/\Delta t$ measured data points: $S(\Delta t), S(2\Delta t), S(3\Delta t), \ldots, S(N\Delta t)$
- Spatial resolution in x-direction Δx is given by the sampling theorem: $\Delta x = FOV / N = 2\pi / (\gamma\ G_x\ N\ \Delta t)$
- With FOV: the maximum object diameter (Field of View), N: number of sampling points, G_x: gradient strength, Δt: sampling interval
- Example: with $N = 256$, $\Delta t = 30\ \mu s$, $Gx = 1.566\ mT/m$ the spatial resolution in x-direction (pixel resolution in x) is: $\Delta x = 1.953\ mm$ and $X = N\ \Delta x = 50\ cm$ (= field-of-view FOV)

Source: Siemens, Magnete, Spins und Resonanzen

Frequency Encoding: Gradient Echo



Signal Equation with spatial frequencies:

$$k(t) = \gamma \int_0^t G(\tau) d\tau$$

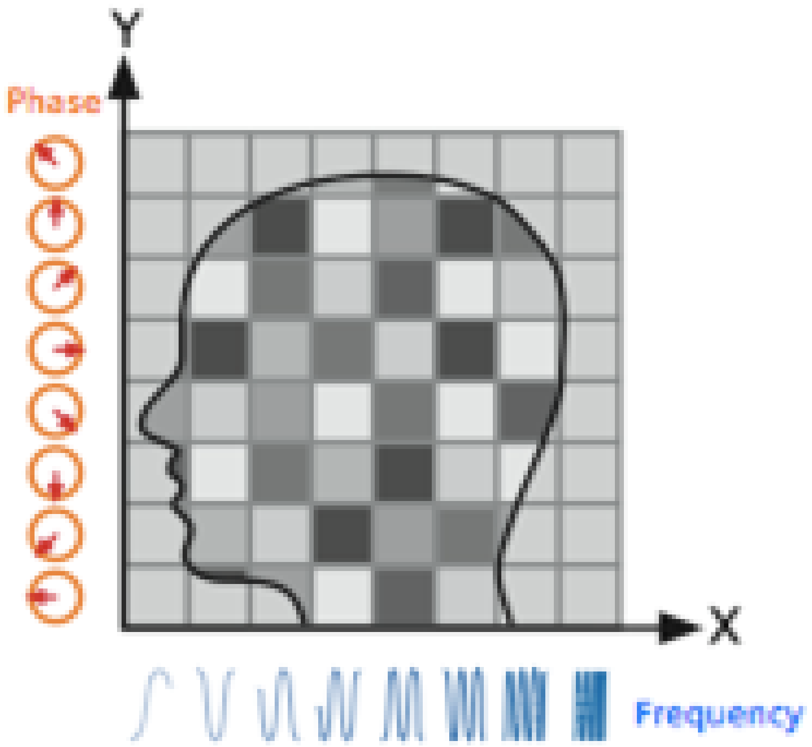
$$S(t) = \int_{object} M_0(x) e^{-ik(t)x} dx \equiv S(k) = FT\left[M_0(x)\right]$$

$$M_0(x) = FT^{-1}\left[S(k)\right] = \int_{-\infty}^{\infty} S(k) e^{ik(t)x} dk$$

- Ideally, *S*(*k*) is Hermitian and therefore knowledge of *S* for *k*>0 is sufficient
- In reality *S*(*k*) contains phase-errors and the acquisition of positive and negative spatial frequencies increases the SNR (Signal-to-Noise-Ratio)

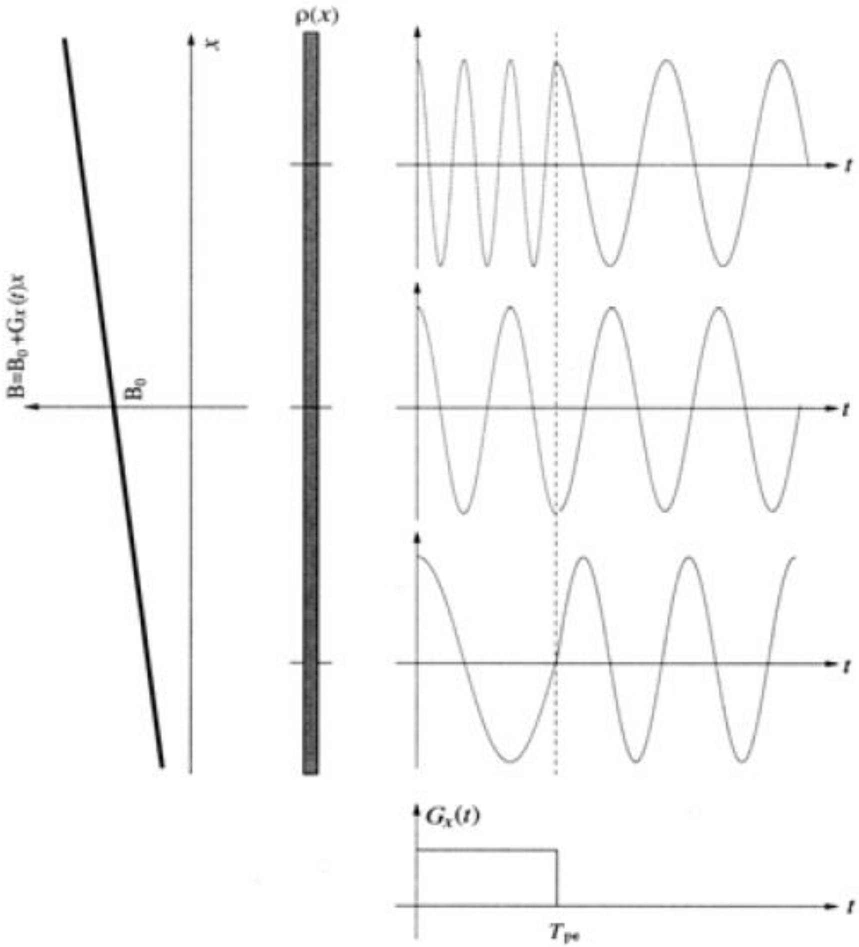
Phase Encoding

- Frequency encoding is applied to one spatial dimension, e.g. the x-axis. How to encode the y-axis?
- Switch on the y-gradient for a short time in order to modulate the spins’ phase in y direction.
- *Repetition of the process with linearly varying phase* again encodes a frequency!



Source: Siemens, Magnete, Spins und Resonanzen

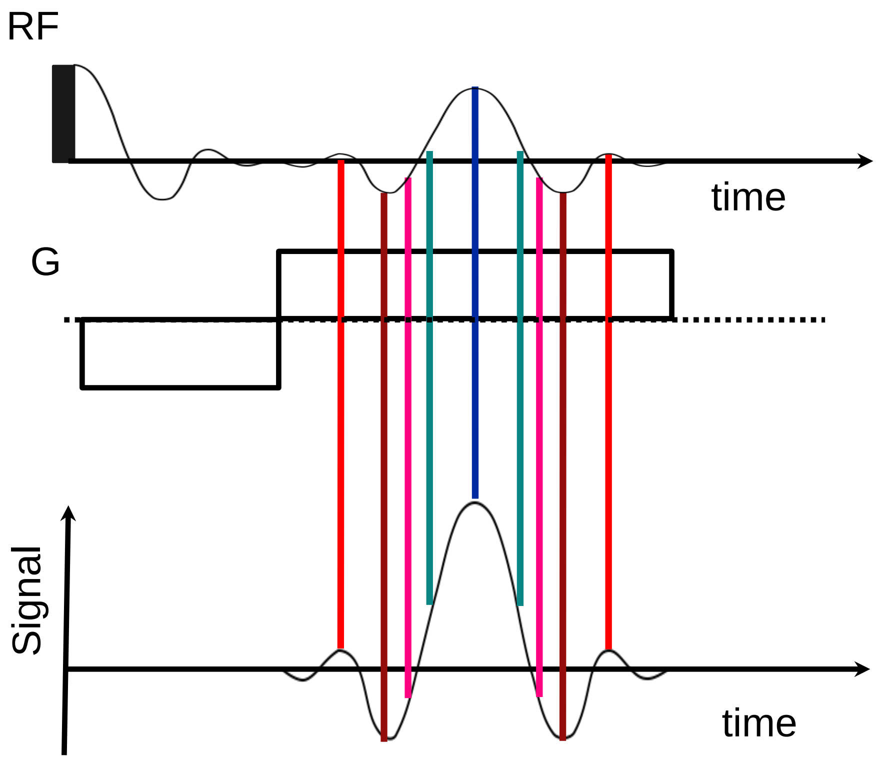
Phase Encoding

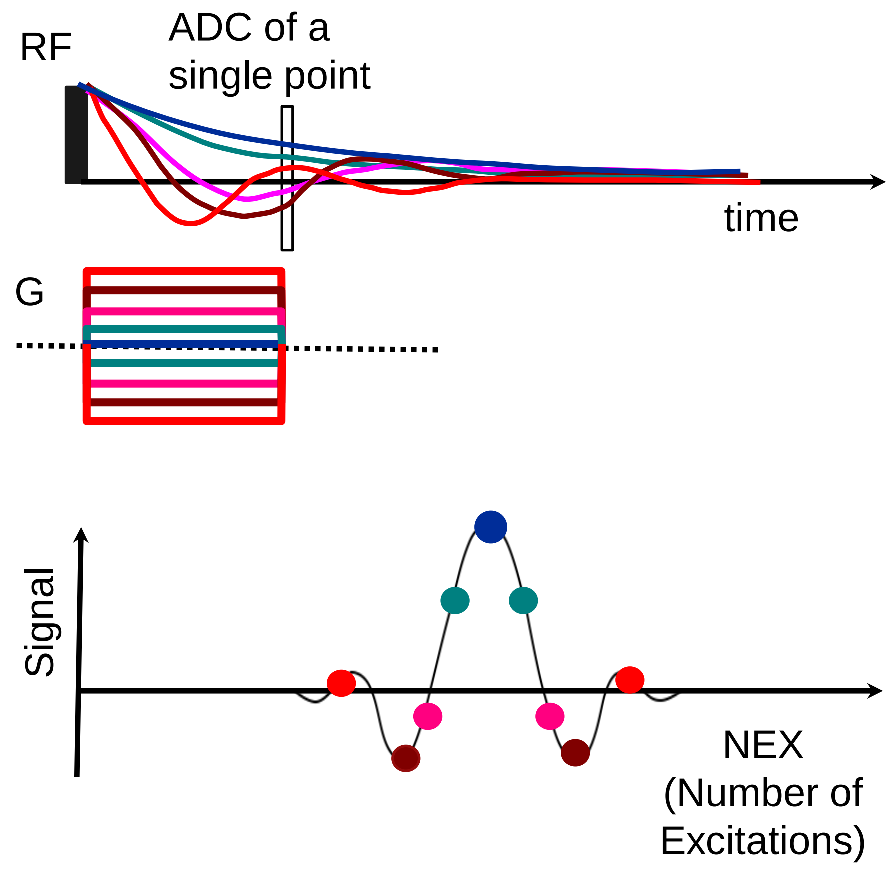


- phase encoding is performed by stamping an initial phase angle onto the excited spins
- after switching off the phase encoding gradient the magnetization is continuing to precede at the same frequency ω_0 but with different phase
- phase information of an activated MR-signal is linearly dependent on spatial coordinate, since $\Phi(y) = - \gamma\ G_y\ y\ T_{pe}$

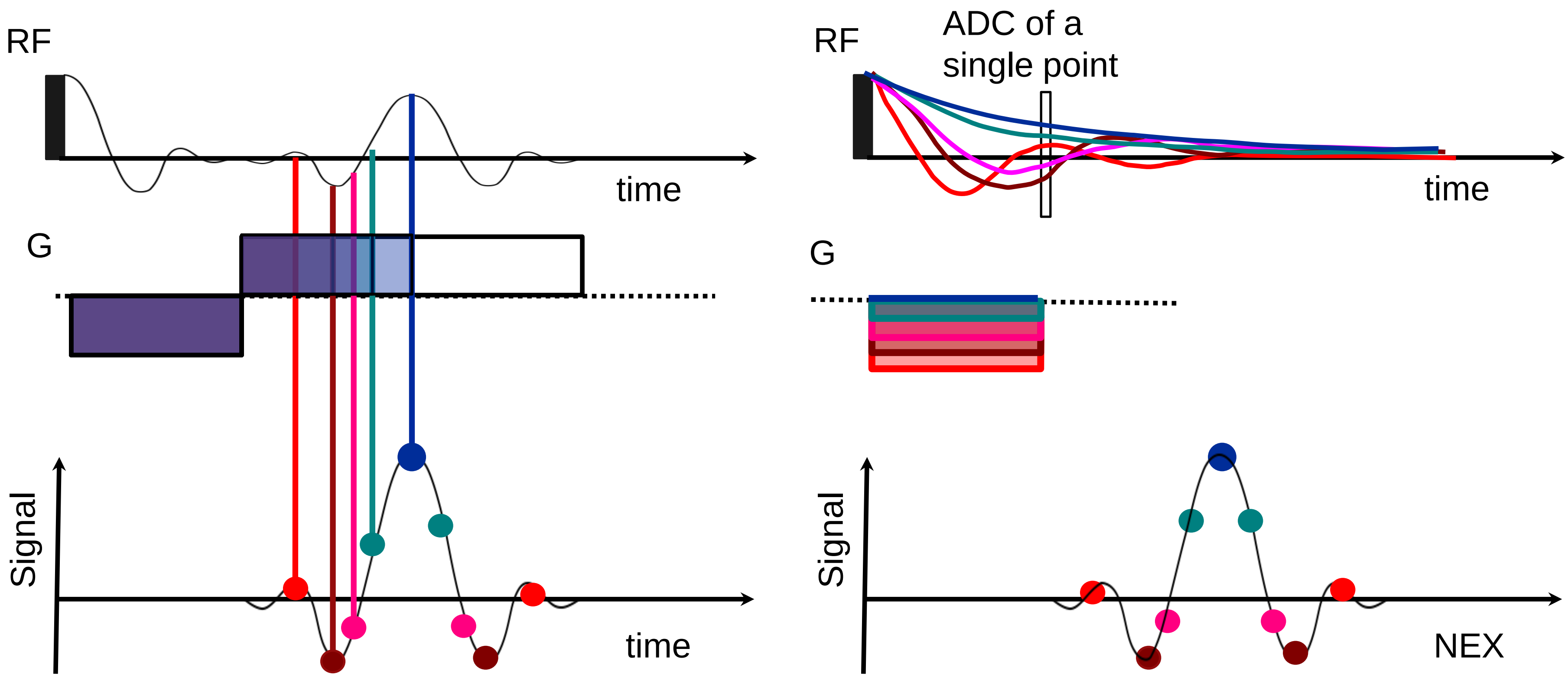
Source: Diplomarbeit Miriam Kubach

Frequency Encoding vs. Phase Encoding

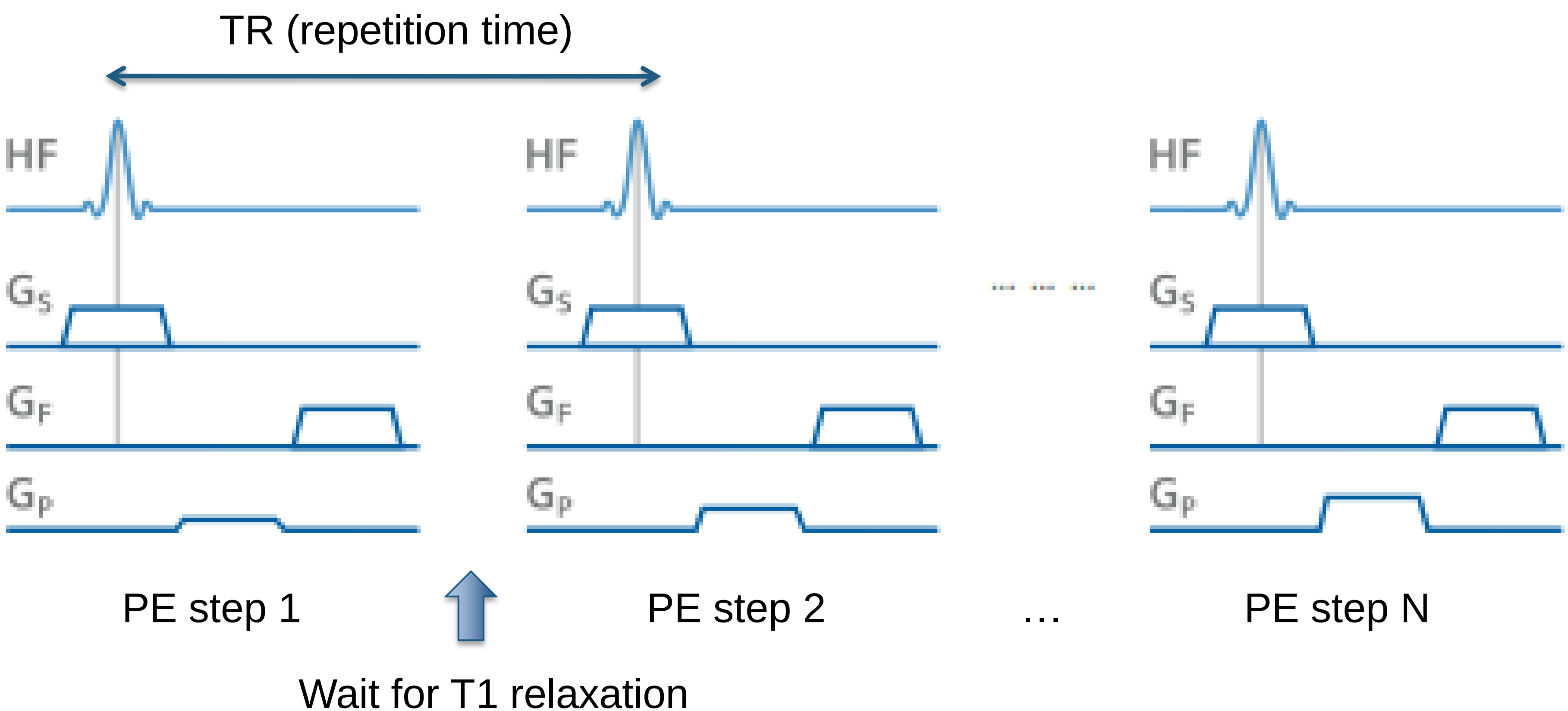




Frequency Encoding vs. Phase Encoding



Timing for Phase and Frequency Encoding



The total acquisition time is given by the repetition time, TR, times the number of phase encode steps!

Source: Siemens, Magnete, Spins und Resonanzen

2D FT-Imaging: Frequency and Phase Encoding

General 2D Fourier Transform Signal Equation:

$$S(k_x,k_y) = e^{-i\omega_0 t} \iint M_0(x,y)\, e^{-ik_x x} e^{-ik_y y}\, dxdy$$

x-Sampling: Frequency Encoding

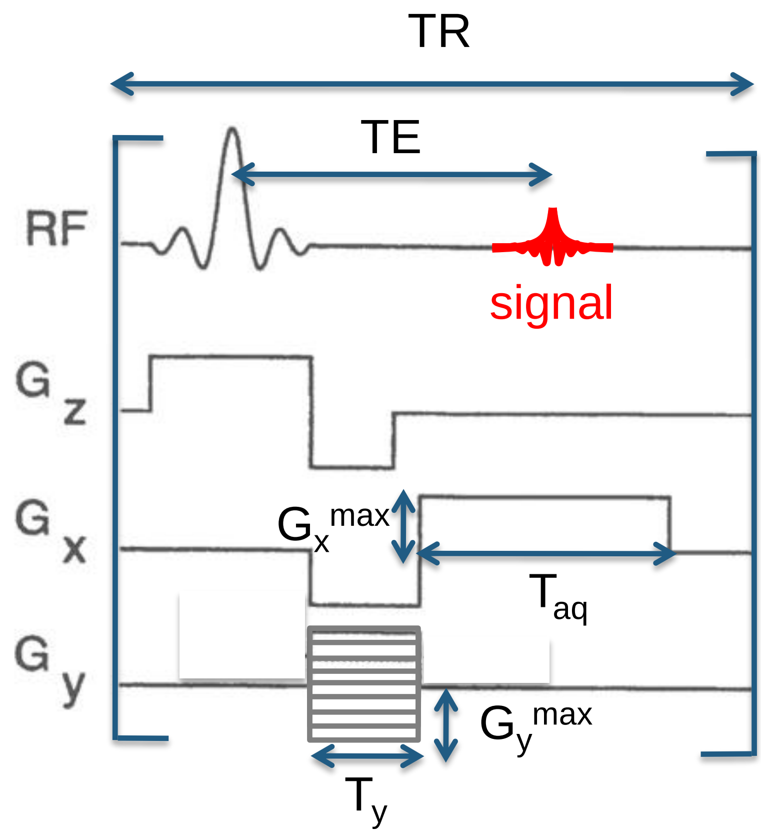
$$k_x(n\Delta t) = \gamma \int_0^{n\Delta t} G_x(\tau)\, d\tau = \gamma G_x^{\max}\left(-\tfrac{1}{2} T_{aq} + n\Delta t\right)$$

$$\Delta t = T_{aq}/N,\; n = 1,\ldots,N$$

y-Sampling: Phase Encoding

$$k_y(mTR) = \gamma \int_{(m-1)TR}^{mTR} G_y(\tau)\, d\tau = \gamma T_y\left(-G_y^{\max} + m\Delta G_y\right)$$

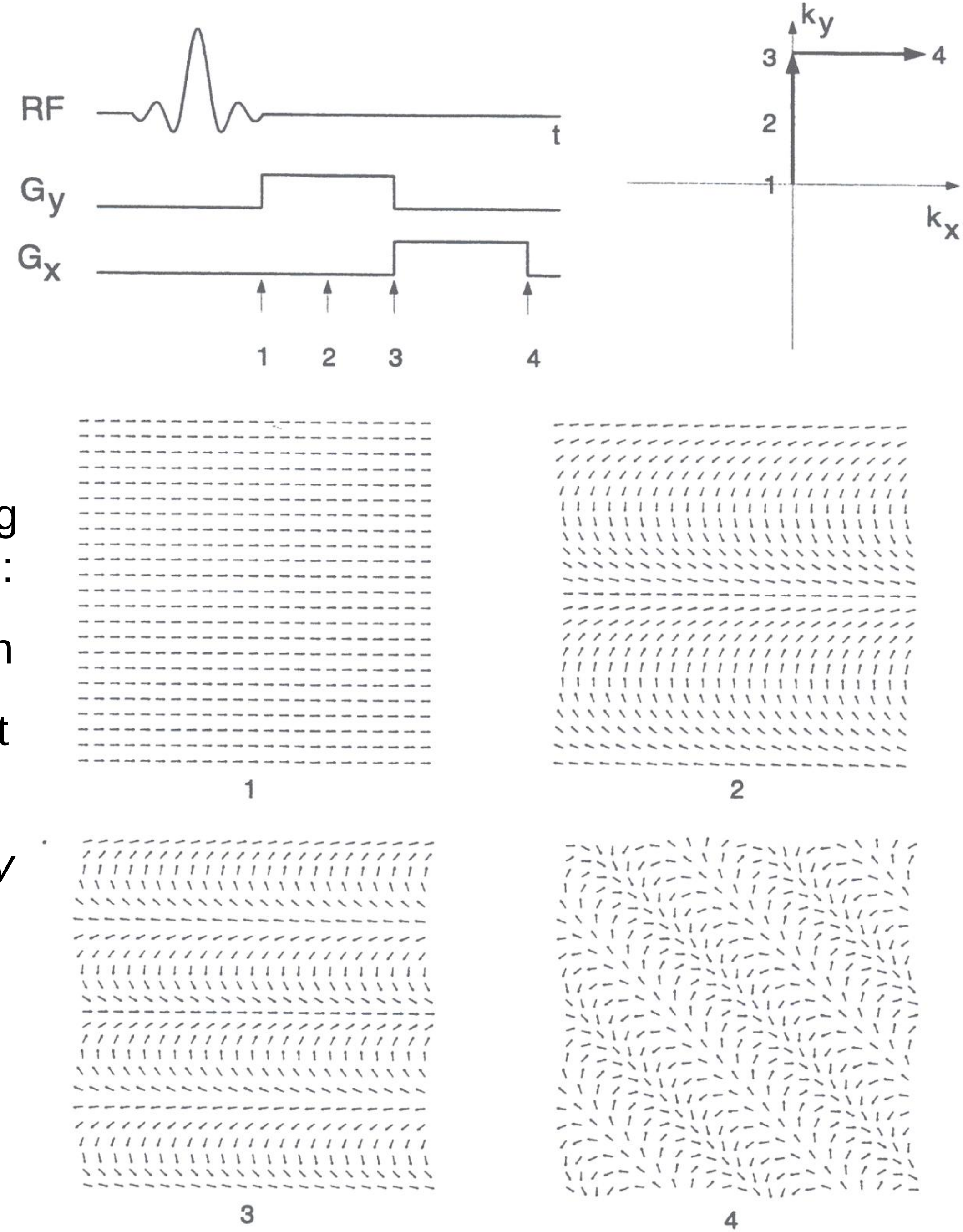
$$\Delta G_y = G_y^{\max}/M, \quad m = 1,\ldots,M$$



- N×M signal Matrix $S(k_x,k_y)$
- 2D-FT gives image $M_0(x,y)$

Source: Siemens, Magnete, Spins und Resonanzen

Encoding 2D spatial Frequencies



Spin distribution corresponding to the four labelled time points:

(1) immediately after excitation

(2) midway after the *y* gradient has been turned on

(3) just prior to turning off the *y* gradient;

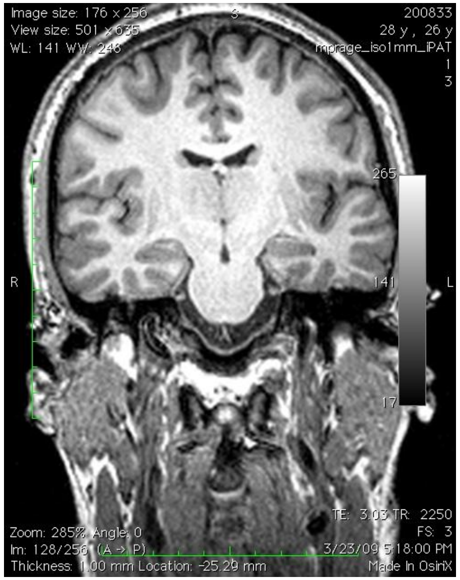
(4) after the *x* gradient has been left on

Note on Oblique Slices

- Gradient encoding can be applied in arbitrary orientation through superposition of physical gradients G_x, G_y, G_z

axial slice

sagittal slice



coronal slice

Note on Oblique Slices
Logical and Physical Gradient System

Logical Gradient System

- Slice-Select Gradient: G_S
- Frequency-Encoding Gradient: G_R (also called Readout gradient)
- Phase-Encoding Gradient: G_P

Transform from logical coordinate system to physical coordinate system with rotation matrices:

$$\begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} = \mathbf{R}_{\phi}(\alpha) \begin{pmatrix} G_R \\ G_P \\ G_S \end{pmatrix}$$

The true physical orientation of the slice does not change the physics, and therefore we are dealing mainly with logical coordinates. Often, $G_x=G_R$, $G_y=G_P$, $G_z=G_S$ is used in textbooks.

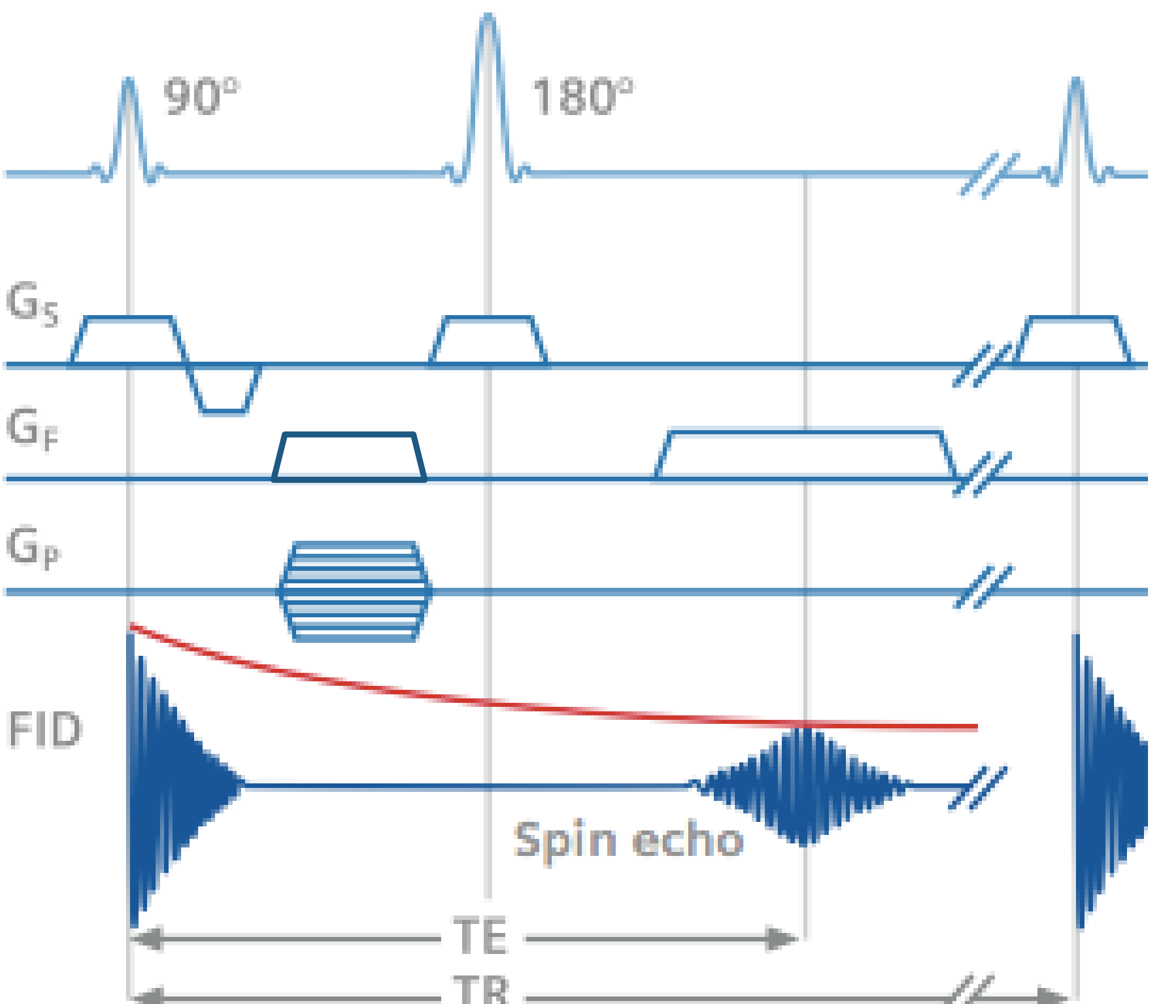
True or False?

1. Frequency Encoding Resolution is limited by long T1
2. Frequency Encoding Resolution is limited by short T1
3. Frequency Encoding Resolution is limited by long T2
4. Frequency Encoding Resolution is limited by short T2
5. Phase Encoding Resolution is limited by long T1
6. Phase Encoding Resolution is limited by short T1
7. Phase Encoding Resolution is limited by long T2
8. Phase Encoding Resolution is limited by short T2

… and what kind of limitations are these?

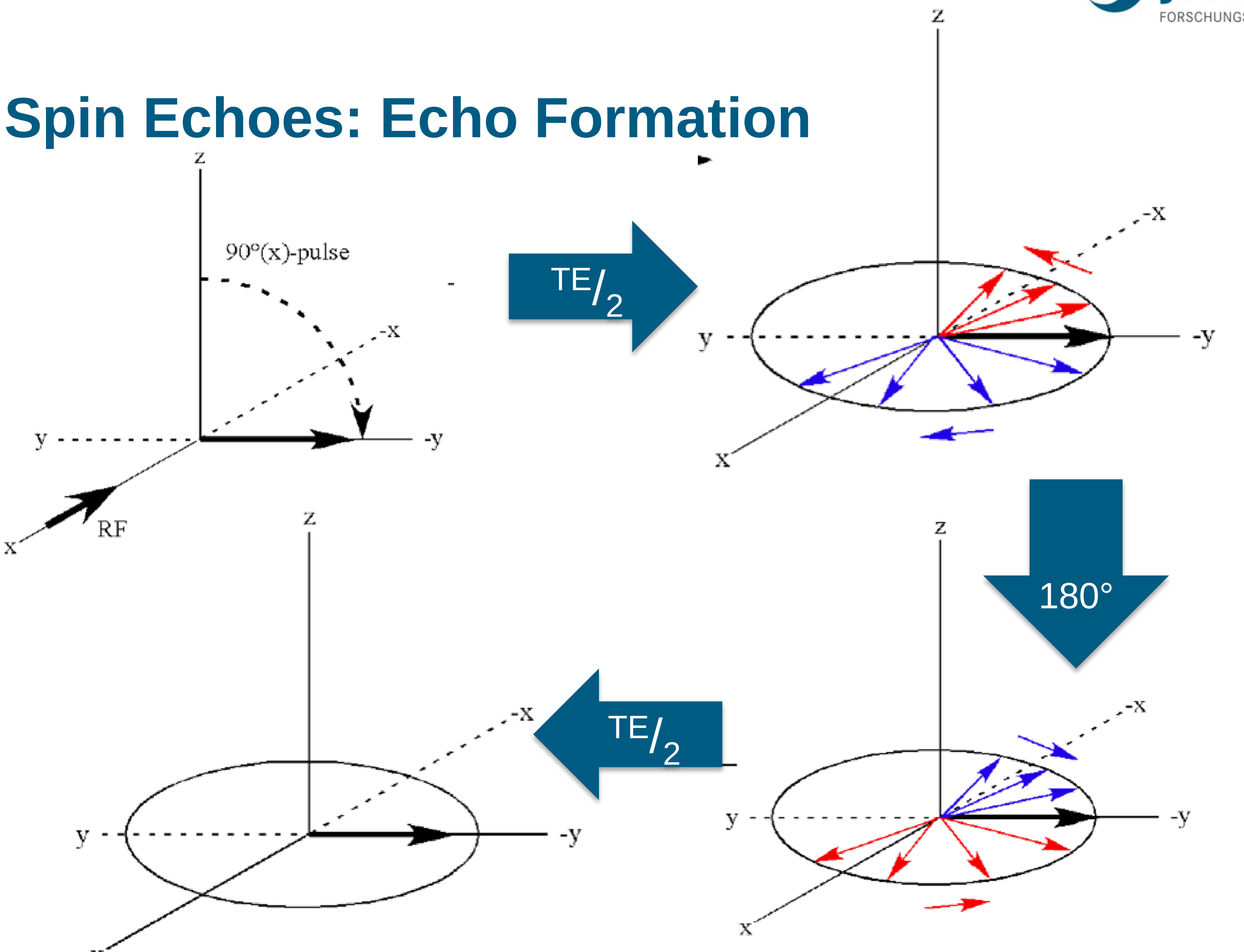
Source: Siemens, Magnete, Spins und Resonanzen

Imaging in Praxis: Spin Echo Sequence



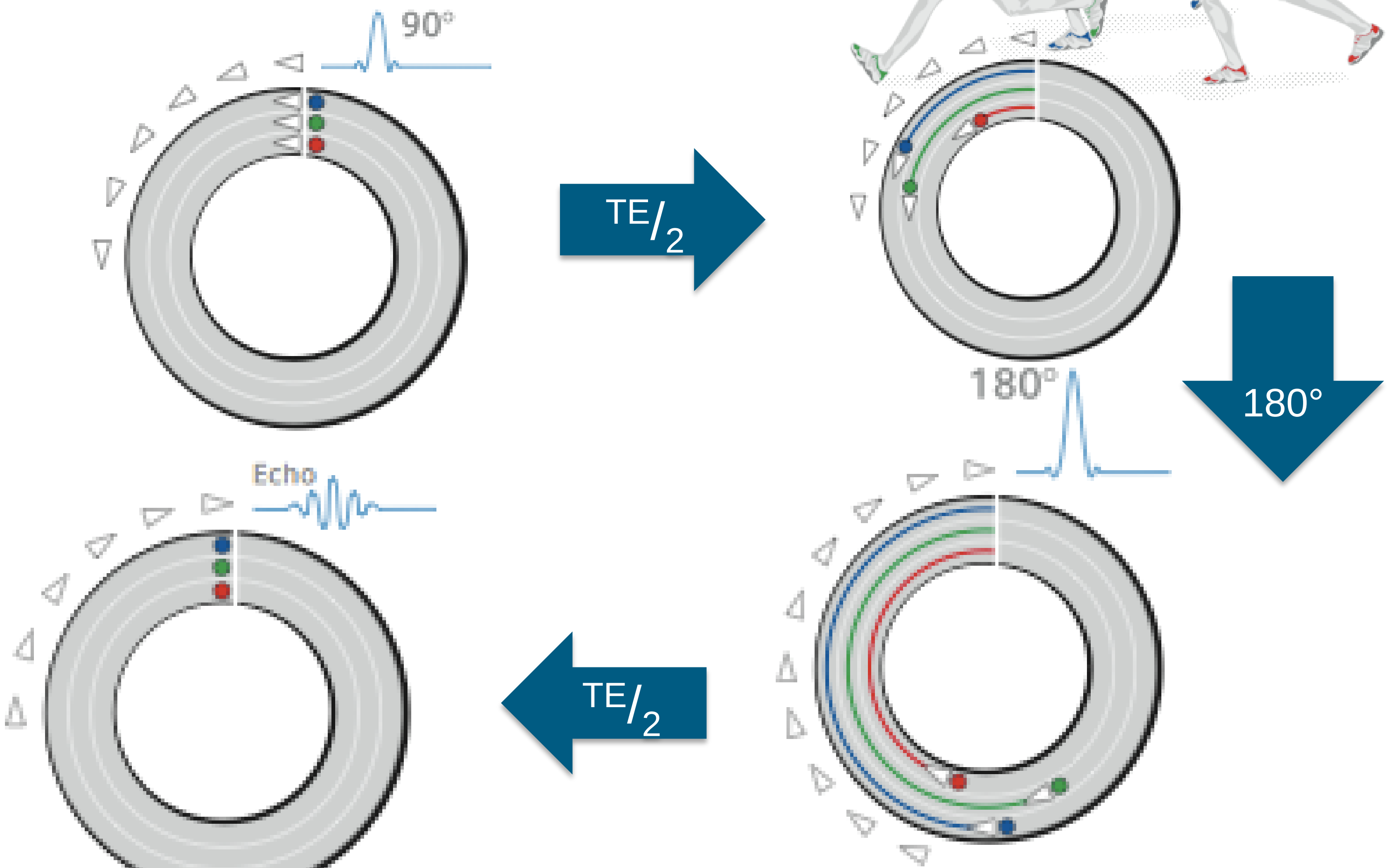
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echoes: Echo Formation



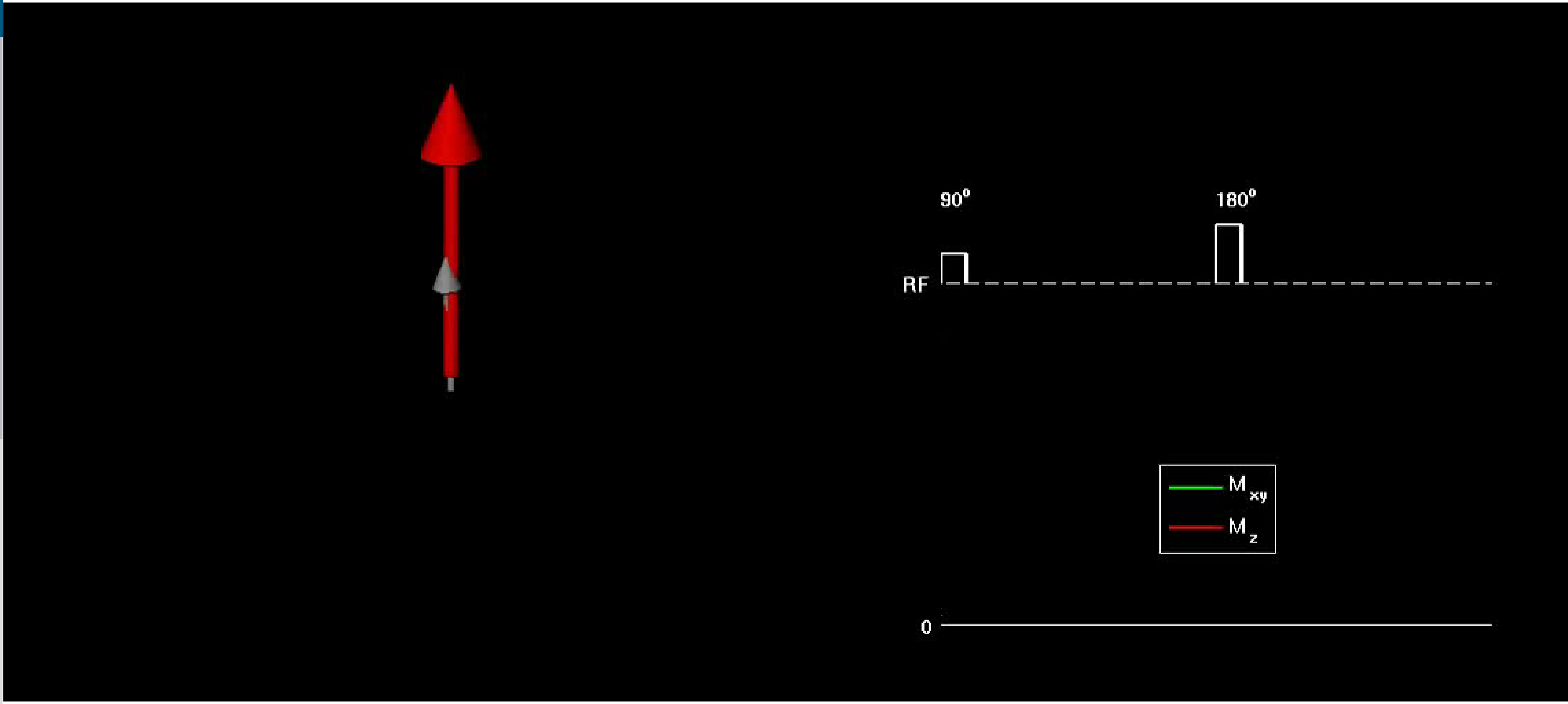
Source: NMR Relaxation Basics, Peter F. Flynn

Spin Echoes: Runners Analogy



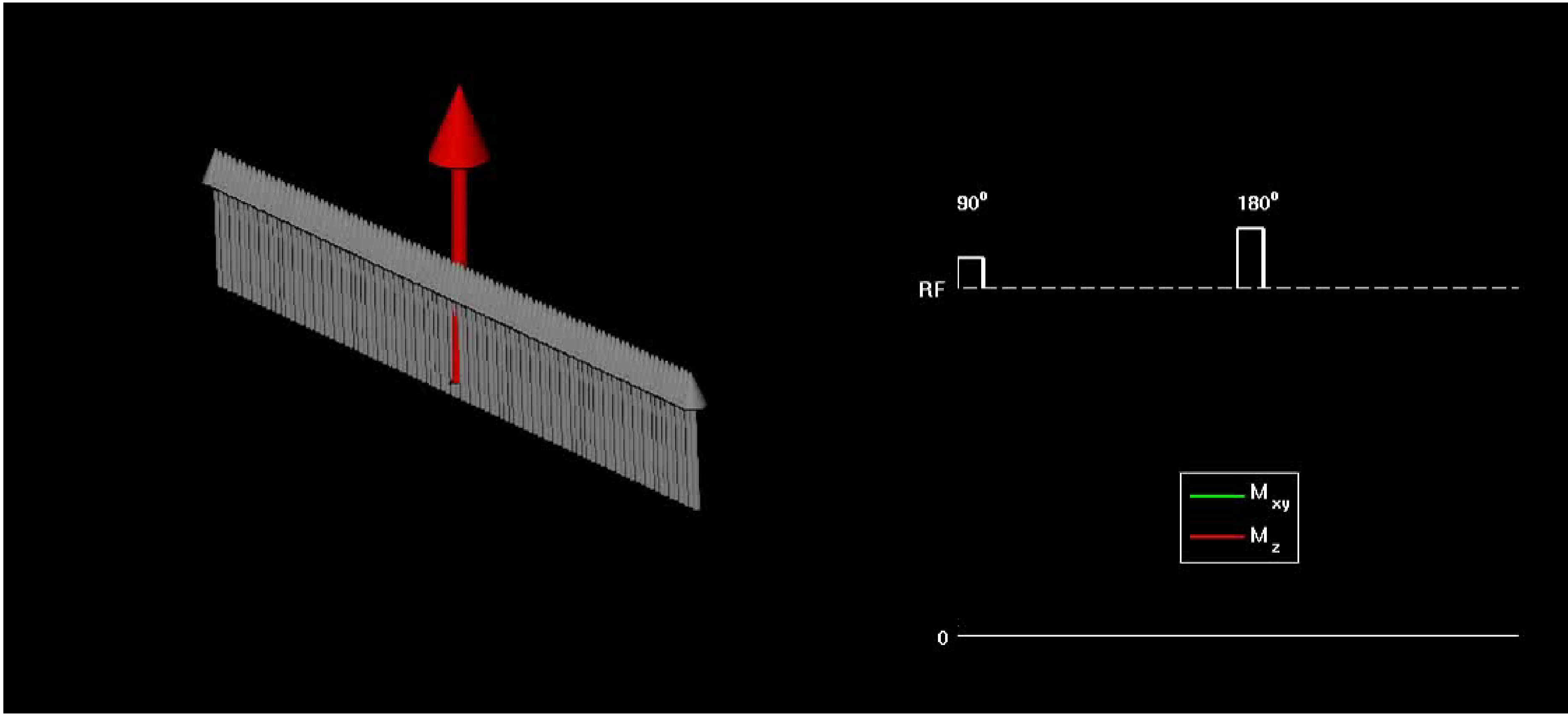
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echoes: in Motion



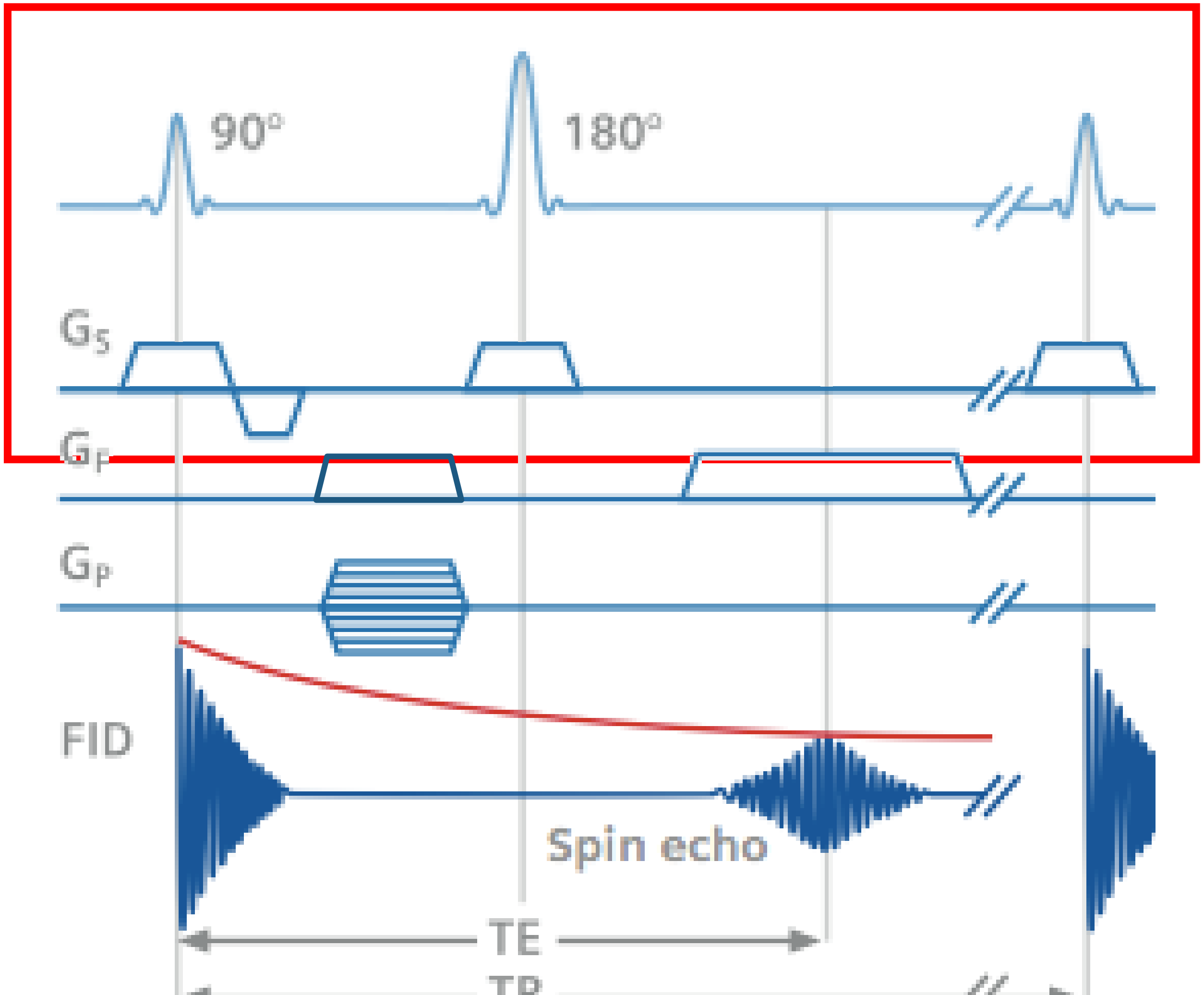
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echoes: in Motion



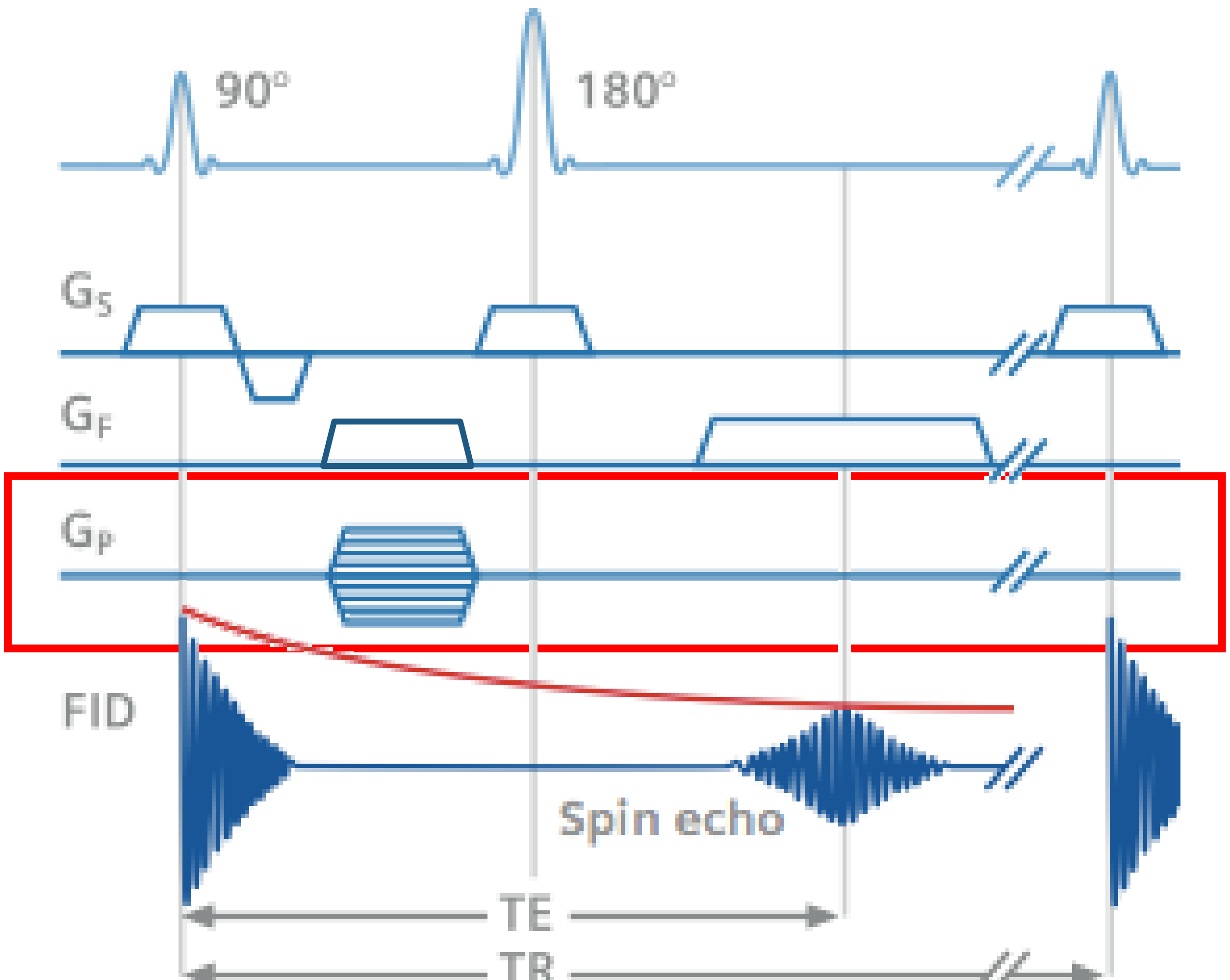
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Slice Selection



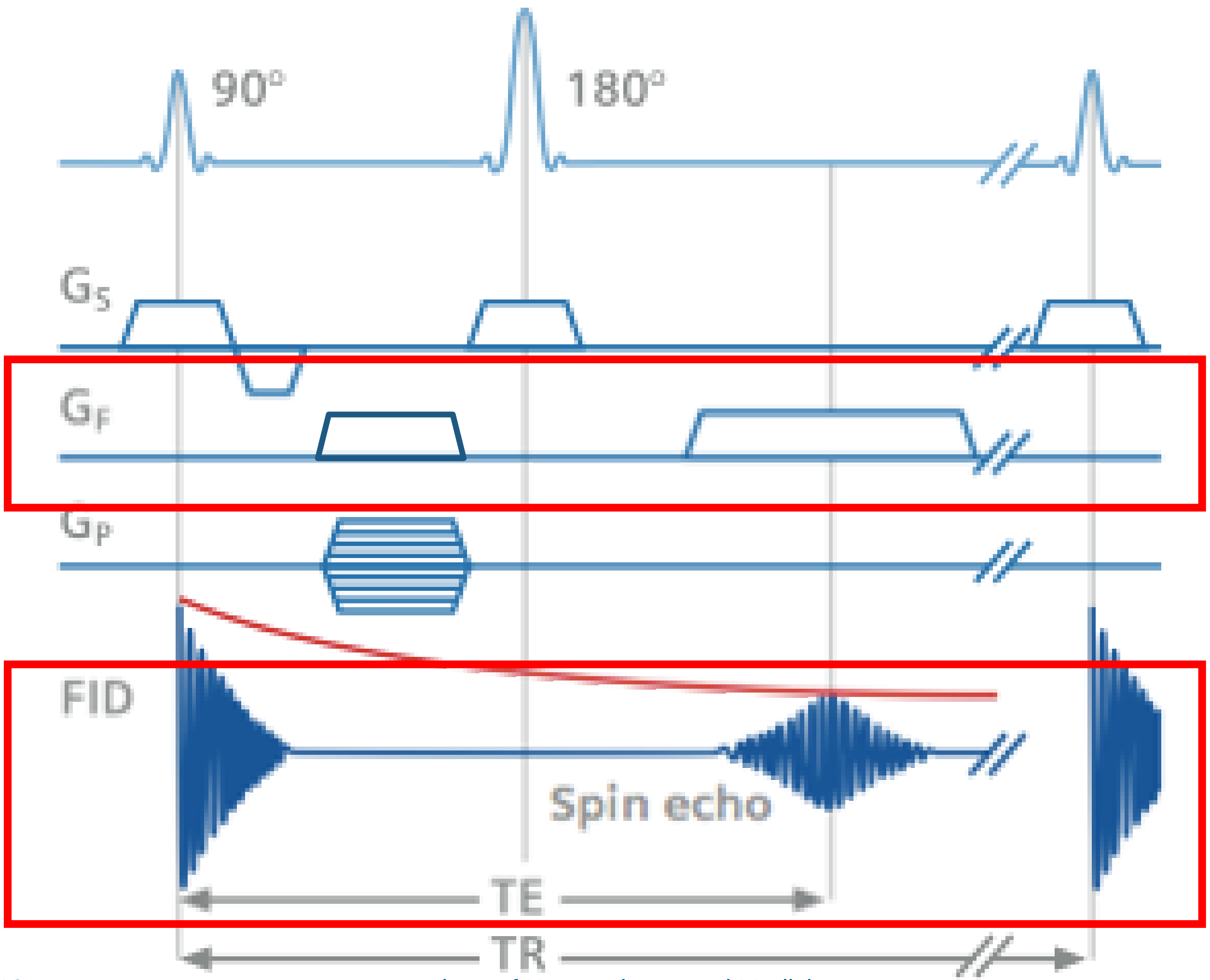
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Phase Encoding



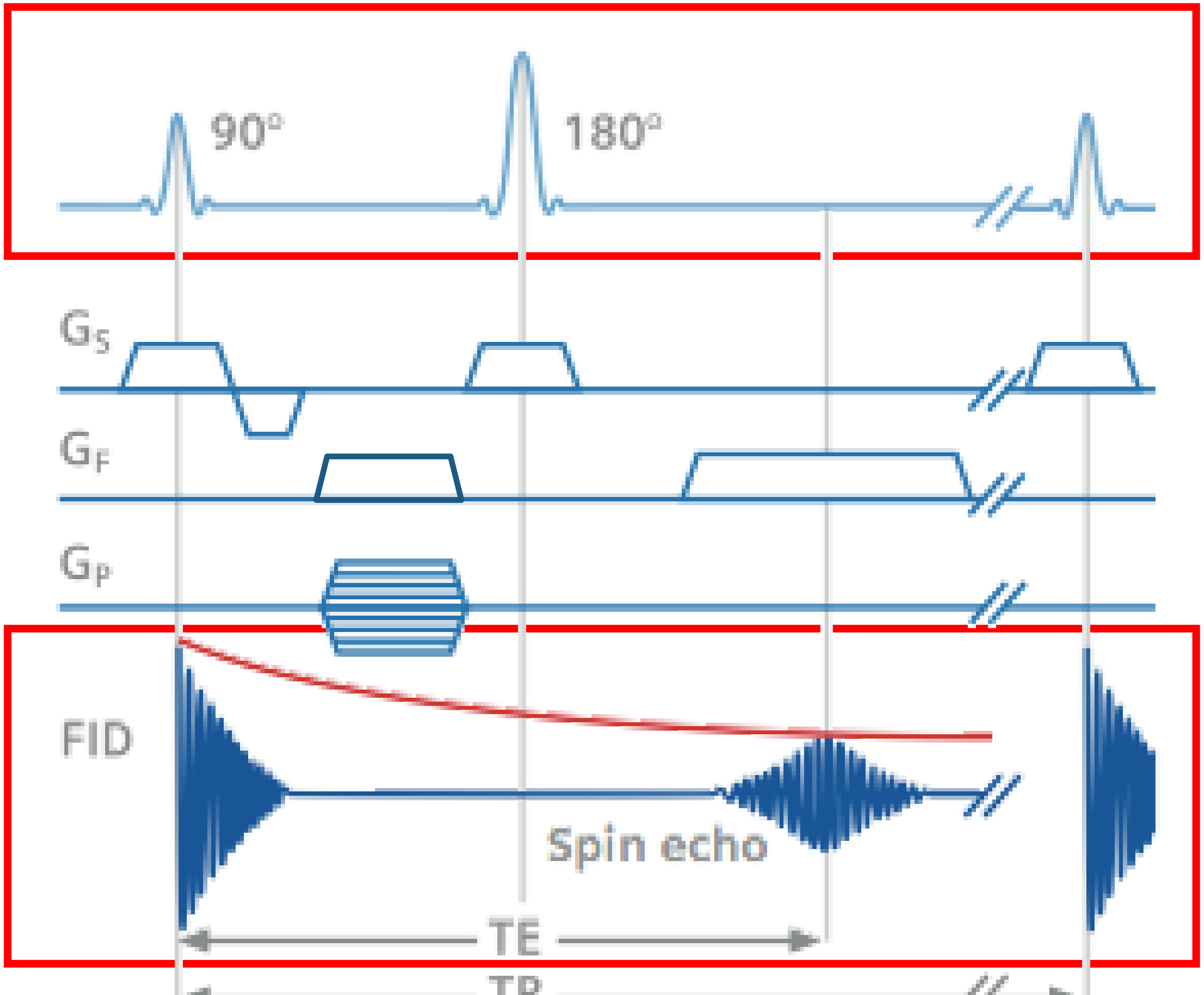
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Frequency Encoding



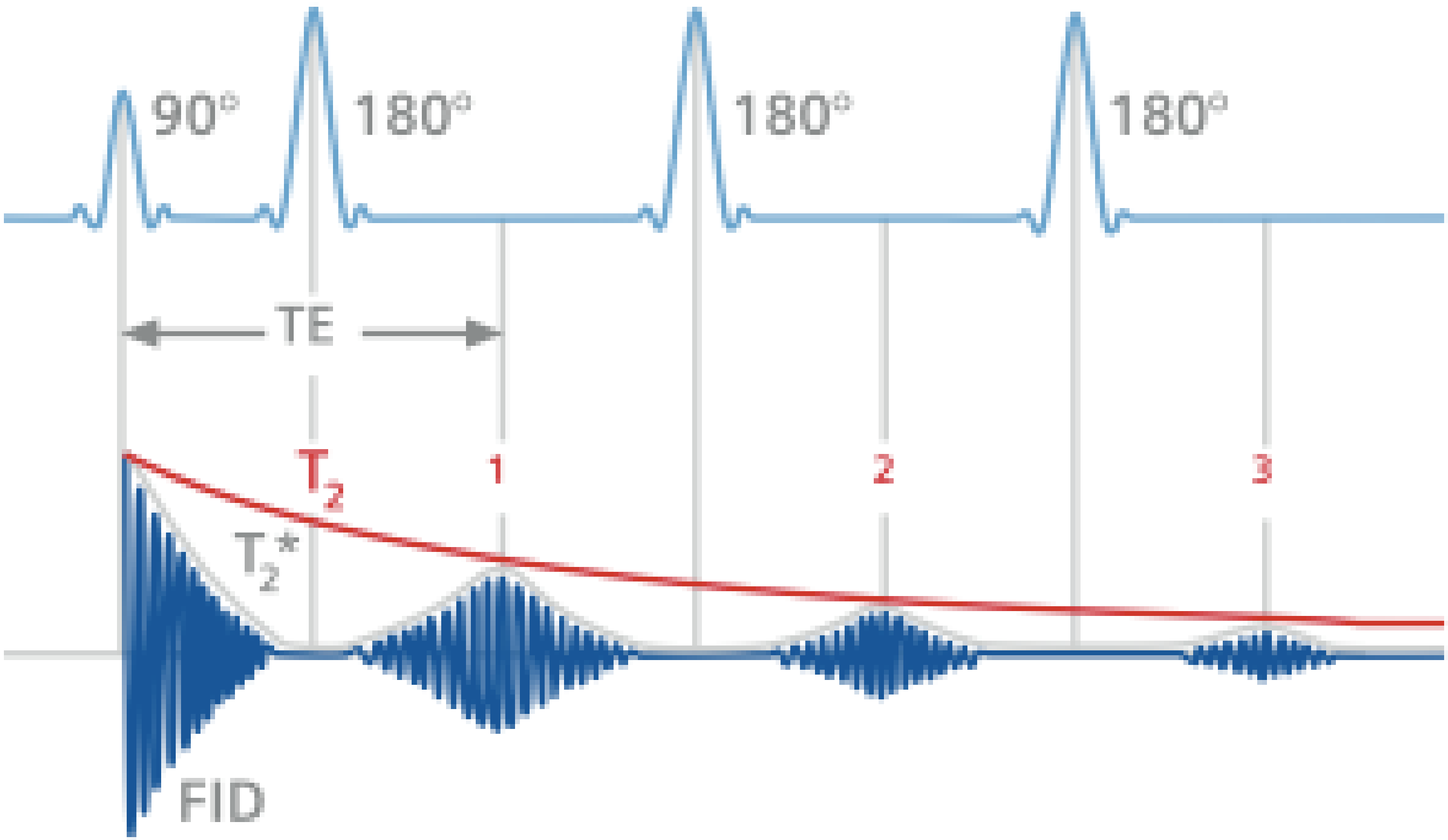
Source: Siemens, Magnete, Spins und Resonanzen

Spin Echo Sequence: Echo Generation



Source: Siemens, Magnete, Spins und Resonanzen

Multiple Spin Echoes: T_2 and T_2*



Source: Siemens, Magnete, Spins und Resonanzen

Summary

- MR Imaging uses **gradients** for a linearly spatial variation of the main field => spatially dependent Larmor frequeny
- **Frequency Encoding**: after slice selection, one dimension within the slice is position-encoded by a *gradient pulse during data acquisition*
- **Phase Encoding:** For the remaining dimension, the excitation has to be *repeated where each time a linearly varying phase-shift is encoded*.
- MRI signals are **echo** acquisitions. Both, gradient echoes or spin echoes are possible.