Mitglied der Helmholtz-Gemeinschaft

JÜLICH FORSCHUNGSZENTRUM

INTERACTION of
RADIATION and PARTICLES with MATTER

Detlev Gotta

Institut für Kernphysik, Forschungszentrum Jülich / Universität zu Köln

“Autumn Lectures” – Nuclear and Medical Physics

Tbilisi, Georgian Technical University (GTU)

October 8-9, 2015

OVERVIEW

BASICS

INTERACTION of ELECTROMAGNETIC RADIATION

INTERACTION of MASSIVE PARTICLES

APPENDIX

BASICS

STRUCTURE of MATTER

PARTICLES and ELECTROMAGNETIC RADIATION

ELECTROMAGNETIC INTERACTION

CROSS SECTION

STRUCTURE of MATTER I

Dalton: Each substance is composed of chemical <u>elements</u>.

1 Mol Na + 1 Mol Cl -> 1 Mol NaCl

One Mol contains always the same number of particles $N_A = 6 \cdot 10^{23}$

The ratios of molar masses of the elements are almost ratios of integer numbers.

Atomic mass unit (a.m.u.)

$$\boxed{1\,\mathbf{a.m.u.} = \frac{\mathbf{m}(^{12}\mathbf{C\,atom})}{12} = \frac{12\,\mathbf{g}}{12 \cdot \mathbf{N_A}} = 1.66\,10^{-27}\,\mathbf{kg}}$$

Periodic system of elements

Mendelejev: Ordering scheme according to chemical properties

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	...	...	...						
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

STRUCTURE of MATTER II

Discovery of the electron (J.J. Thomson 1897)

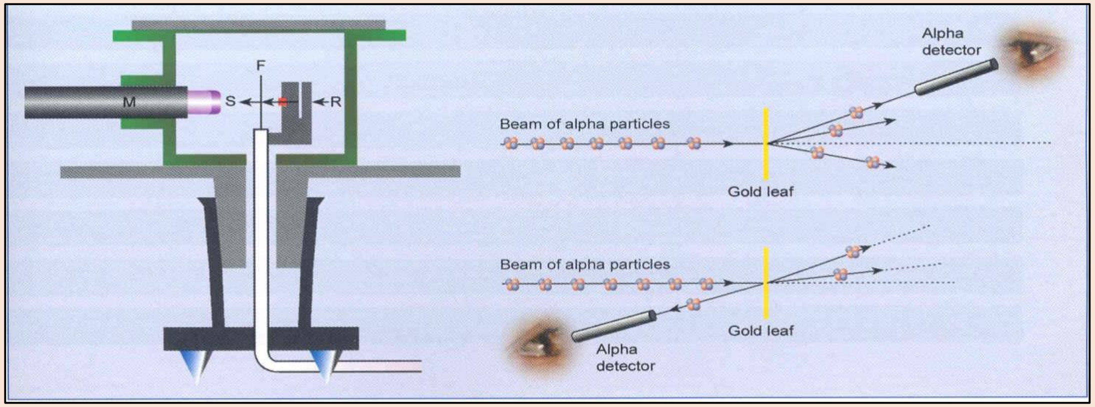
Interpretation of the rays accompanying gas discharges:
Kathode rays: Electrons (negatively charged and light)
Channel rays: Ions (positively charged and heavy)

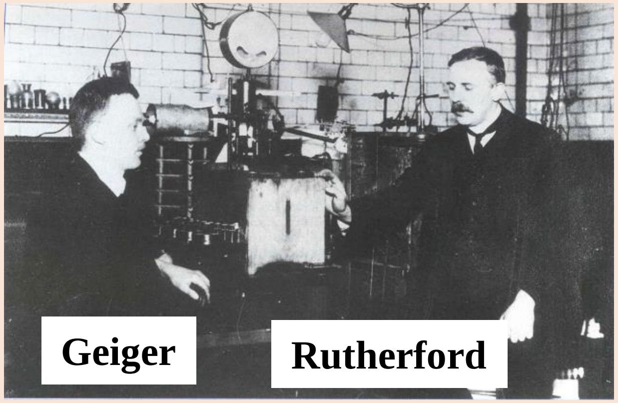
$$m_e = \frac{1}{1836} \cdot m_u, \; q_e = -e$$

Discovery of the atomic nucleus (Geiger & Rutherford 1911)

Almost the full mass of an atom of diameter ~ 10^{-10} m is concentrated in a tiny volume of radius ~ 10^{-15} m

Proof: Collision kinematics – backscattering <u>only</u> from heavy collsion partners





$\Rightarrow$ *total atom is electrical neutral: Atomic nucleus - electric charge* $+Z\,q_e$ *Z whole number*
Atomic shell - electric charge $-Z\,q_e$

STRUCTURE OF MATTER III

Bohr-Sommerfeld model of atoms

main quantum number $n = 1, 2, \ldots$

"main shell"

angular momentum $\ell = 0, \ldots, n-1$

„sub-shell"

magnetic quantum number $m = -\ell, -\ell+1, \ldots, \ell-1, \ell$

($2\ell+1$) possible orientations of angular momentum vector in external field

intrinsic electron spin $S = ½$

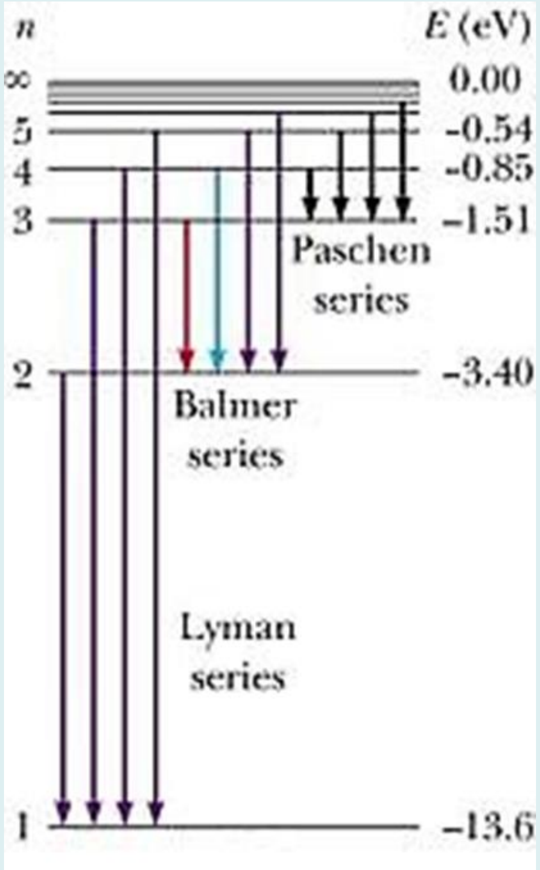
intrinsic angular momentum

2 possible orientations of spin vector in external field $S = \pm ½$

total spin: $\vec{j} = \vec{\ell} + \vec{S}$ $\quad |\vec{j}| = \frac{1}{2}, 1, \frac{3}{2}, \ldots$

ℓ and S are measured in units $\hbar = \frac{h}{2\pi}$ (h: Planck constant)

main shells for Z = 1 (H)



STRUCTURE of MATTER IV

S = ½ ***particles are called fermions***

Pauli principle:

Only one fermion is allowed in a particular quantum state: For atoms = (n, ℓ, m, S)

$\Rightarrow$ ***maximum no. of electrons per sub-shell:*** $2\cdot(2\ell + 1)$

Periodic system of elements

Mendelejev: Ordering scheme according to chemical properties

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	...	...	...						

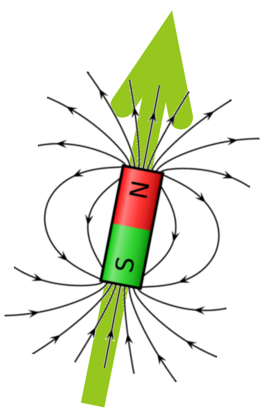
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

outmost unfilled shell determines chemistry

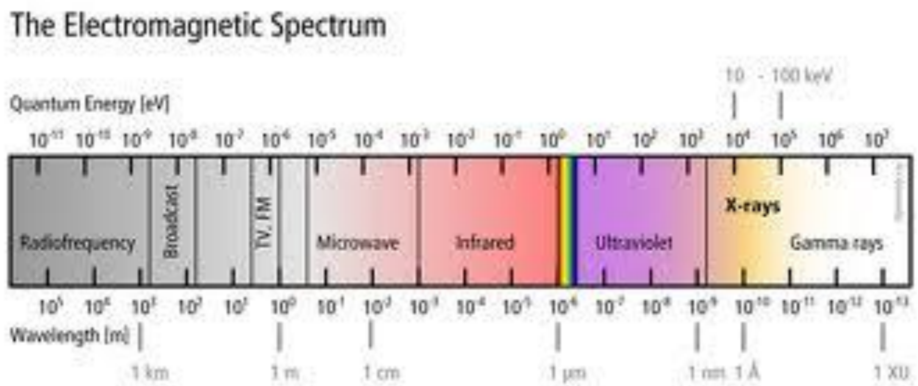
PARTICLES

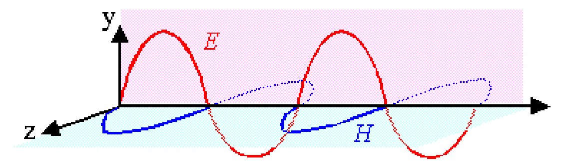
What characterizes a particle?

mass		m
charge		Q
Spin	***intrinsic angular momentum***	S $\Leftrightarrow$ ***magnetic moment*** μ
life time		τ_0
shape	***for non elementary particles***	$\langle r^2 \rangle$
quantum mechanics		$\lambda = \frac{h}{p}$ ***matter waves (de Broglie 1922)***



ELECTROMAGNETIC RADIATION





wave length λ

frequency ν

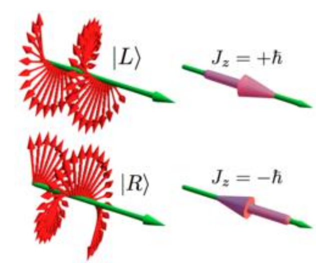
wave propagation velocity in vacuum $c = \lambda\,\nu$

" " " in medium $c' = \lambda'\nu < c$

index of refraction $n = c\,/\,c'$

quantum mechanics: waves can be particles

Photon having spin 1 $m = \pm 1$, ***no*** $m = 0$



left/right circular polarization

having energy $E = h\nu$ ***(Einstein 1905)***

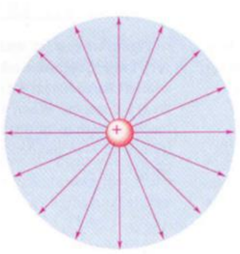
EL.-MAG. INTERACTION I - ELECTRIC FORCE

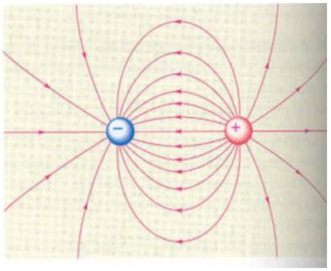
the force is mediated in the

classical picture

by field around a source

$$F_{Coulomb} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$$

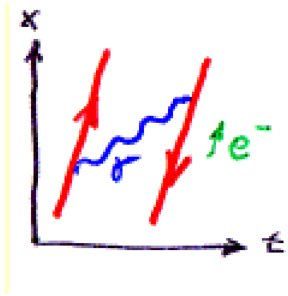




quantum world

field quanta = particles

„light“ particles = photons γ

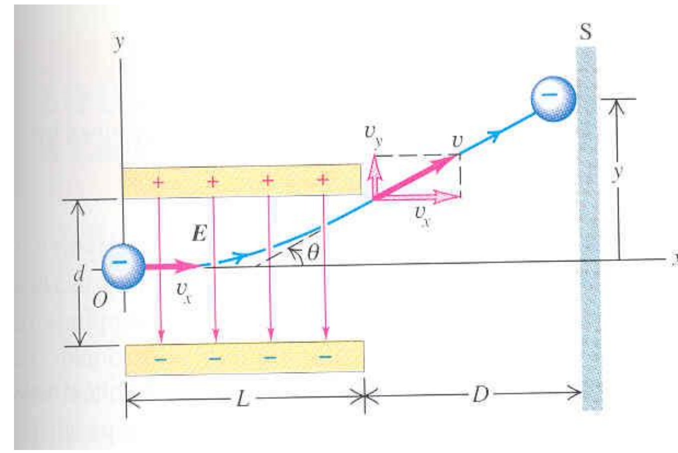


electromagnetic radiation = E and B fields interact with electric charges

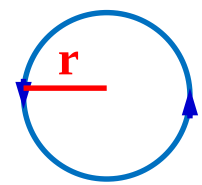
EL.-MAG. INTERACTION II - CHARGES in EL.-MAG. FIELDS

- **electric field** $\vec{F} = m\ddot{\vec{x}} = Q \cdot \vec{E}$

$$\boxed{\Rightarrow \quad \frac{Q}{M}}$$



- **magnetic field** $\vec{F} = m\ddot{\vec{x}} = Q \cdot \left(\vec{v} \times \vec{B}\right)$



B = const.

$\Rightarrow$ *circular motion*

B ⊥ plane of projection

$$\omega = \frac{Q}{M} B \qquad \omega = \frac{2\pi}{T}$$

$$\boxed{\Rightarrow \quad p}$$

$$mv^2 / r = Q \cdot v \cdot B$$

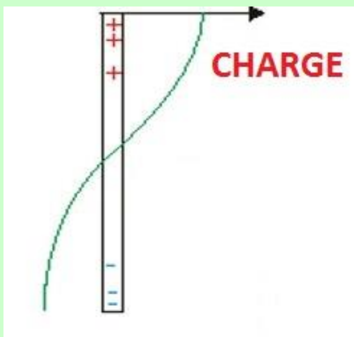
$$p = Q \cdot B \cdot r$$

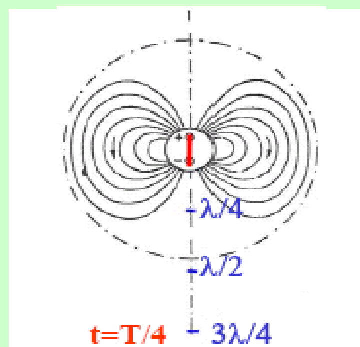
EL.-MAG. INTERACTION III: HERTZIAN DIPOLE

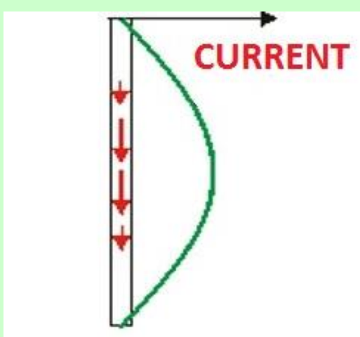
Hertz 1888

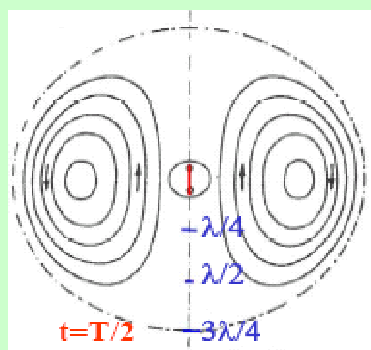
Emitter: *The variable E and H fields are emitted with speed of light forming the transverse el.-mag. wave*

Receiver: *The variable E and H fields of the arriving el.-mag. wave induce currents because of forces on charges*



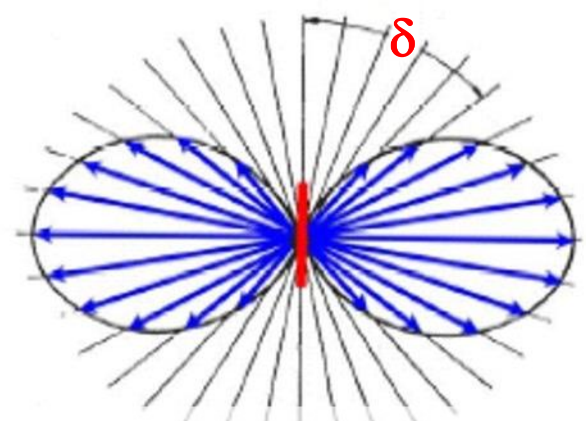






directional characteristics

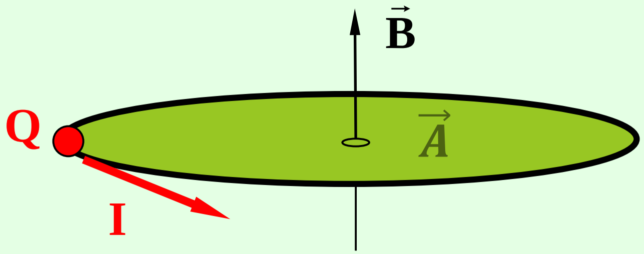
cross section of radiated intensity (time average)



$$\left|\vec{S}\right| \propto \nu^4 \cdot \frac{\sin^2\delta}{r^2} \qquad [S] = \frac{V}{m}\cdot\frac{A}{M} = \frac{W}{m^2}$$

EL.-MAG. INTERACTION IV - MAGNETIC MOMENT

„magneto-mechanical“ view of magnetic moment



magnetic dipole is produced by

circular current $I = Q \cdot \frac{\omega}{2\pi}$

magnetic moment $\vec{\mu} \equiv I \cdot \vec{A} = \frac{Q}{2m} \cdot \vec{L}$

quantum mechanics $L = n\hbar,$ $n = 0, 1, 2, \ldots$

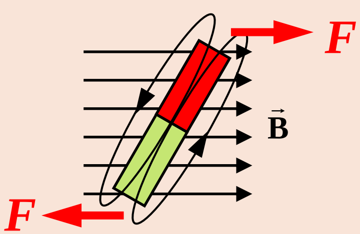
subatomic particles: possible without any mechanical analogon (extension)

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$ $\mu_e = -\, g_e \cdot \mu_B \cdot \frac{S}{\hbar}$ $g_e = 2 + 2 \cdot 10^{-6}$ elementary particle

nuclear magneton $\mu_N = \frac{e\hbar}{2m_p}$ $\mu_p = g_p \cdot \mu_B \cdot \frac{S}{\hbar}$ $g_p = 5.6$ not an elementary particle

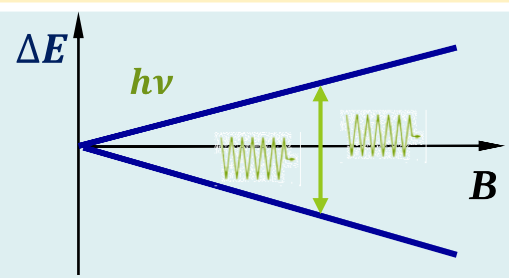
torque $\vec{M} = \vec{\mu} \times \vec{B}$

change in units of $g \cdot \mu_B / g \cdot \mu_N$



interaction energy

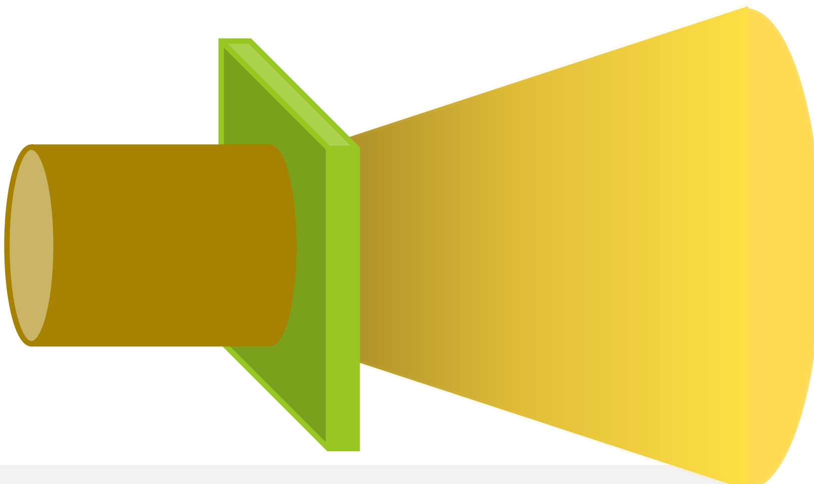
$\Delta E = \vec{\mu} \cdot \vec{B}$



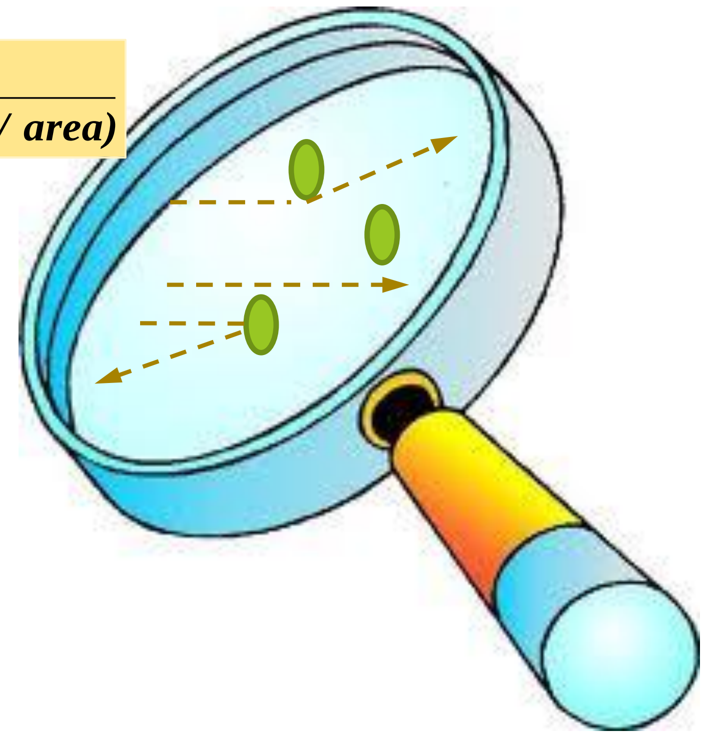
CROSS SECTION I

measures the probability of a reaction

$$\sigma = \frac{\textit{reactions / time interval}}{(\textit{incoming particles / time interval}) \times (\textit{target particles / area})}$$



F = target area $\cap$ beam cross section [cm^2]



particle $\sigma = \frac{N}{N_0} \cdot \frac{1}{n/F}$ *N_0 incoming particles in beam*

wave $\sigma = \frac{I}{I_0} \cdot \frac{1}{n/F}$ *intensity I_0 of incoming radiation*

ρ	= target density	[g / cm^3]
n□	= n / F ×d	
	= $\rho \cdot (N_A / A)$	[particles / cm^3]
N_A	= Avogadro constant	
A	= mass number	[g]

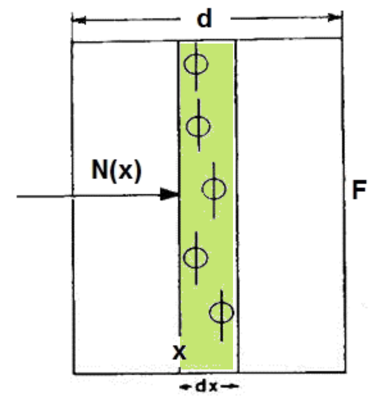
CROSS SECTION II

reaction rate

„thin" target *differential notation (N → dN)*

$$-\,dN(x) = N(x)\cdot\left(\rho\cdot\frac{N_A}{A}\right)\cdot\sigma\cdot dx$$

setup *„physics"*



„thick" target *integration over target thickness* d

$N(d) = N(0)\cdot e^{-\rho\cdot\frac{N_A}{A}\cdot\sigma\cdot d}$ = *number of not interacting particles (transmission)*

$R(d) = N(0)\cdot\left(1 - e^{-\rho\cdot\frac{N_A}{A}\cdot\sigma\cdot d}\right)$ = *number of interacting particles (reaction/absorption)*

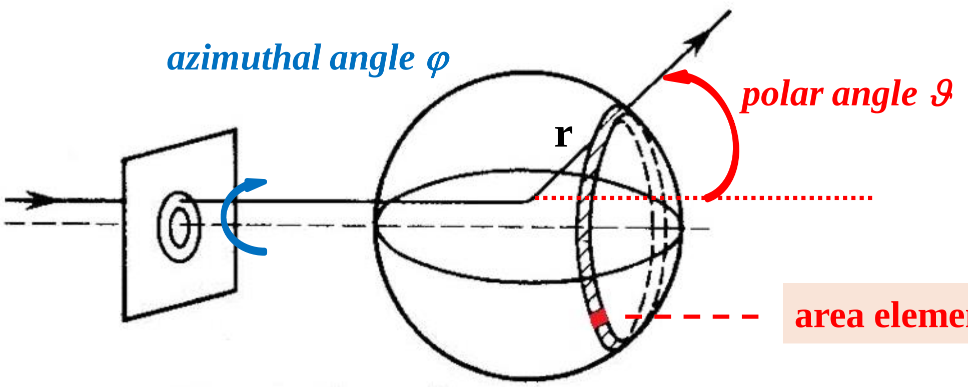
linear extinction coefficient $\mu \equiv (\rho\cdot\frac{N_A}{A})\cdot\sigma$ **ρ: status of target A**

mass absorption coefficient $\lambda^{-1} \equiv \frac{\mu}{\varrho} = \frac{N_A}{A}\cdot\sigma$ **independent of state of aggregation of target A**

mixture/compound **total cross section** $\sigma_{total}(E) = \Sigma\,\sigma_{partial}(E)$ **[1 barn = 10^{-24} cm^2]**

CROSS SECTION III

differential cross section includes angular information of scattering



solid angle element

$$\Delta\Omega \equiv \Delta A / r^2 \quad \text{[steradian (sr)]}$$

area element $\Delta A = r\,\Delta\vartheta \cdot r \sin\vartheta\,\Delta\varphi$

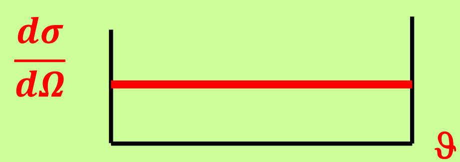
azimuthal symmetry $\Delta\Omega = 2\pi \sin\vartheta\,\Delta\vartheta$ *φ integration*

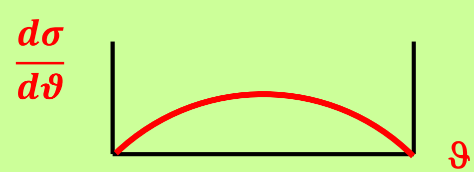
full sphere $\Omega = 4\pi r^2 / r^2 = 4\pi$ („4π“ detector)

fraction of full solid angle $\Delta\Omega / 4\pi$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\pi \cdot \sin\vartheta} \cdot \frac{d\sigma}{d\vartheta}$$

$$\frac{d\sigma}{d\Omega} = const.$$ **isotropy in ϑ**





INTERACTION of ELECTROMAGNETIC RADIATION

SCATTERING

PHOTO EFFECT

COMPTON EFFECT

PAIR PRODUCTION

BREMSSTRAHLUNG & EL.-MAG. SHOWER

ATTENUATION

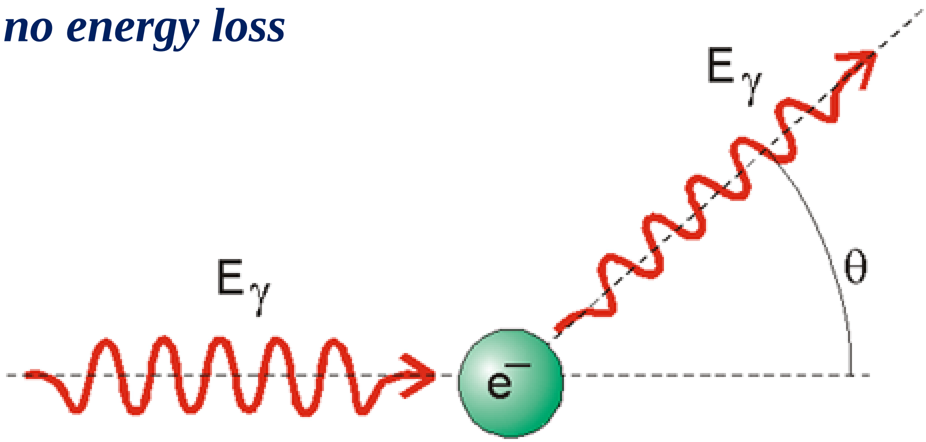
THOMSON SCATTERING

elastic scattering of el.-mag. waves at a free charges = electron, …

independent of wave length λ

$$\boxed{\sigma_{Th,e} = \frac{8\pi}{3} \cdot {r_e}^2 \cong \frac{2}{3}\ barn}$$

no energy loss



$$\sigma_{Th,atom} = Z \cdot \sigma_{Th}$$

deviates from experiment

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = \alpha \cdot \frac{\hbar c}{m_e c^2} = 2.82 \cdot 10^{-15}\ m$$

application: plasma diagnosis, polarization of CMB, …

RAYLEIGH SCATTERING

elastic scattering of el.-mag. waves at polarisable scattering centers = atoms, molecules

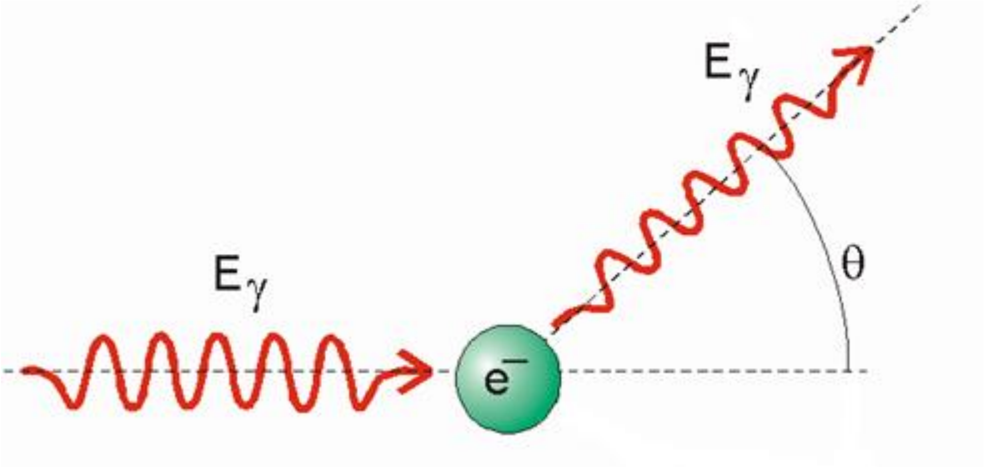
damped oscillation of „elastically“ bound electrons

eigen frequency ω_0

$$\sigma_R = \sigma_{Th} \cdot \frac{\omega^4}{(\omega^2 - {\omega_0}^2)^2} \cdot Z^2$$

ω_0 eigen frequency

no energy loss



$\omega \ll \omega_0$ *makes the sky blue / sunset red*

application: combustion diagnosis, holidays, …

PHOTO EFFECT

requires particle nature of „light“ Einstein 1905

$$\sigma_{2K} = \sigma_{Th} \cdot 4\sqrt{2}\alpha^4 \cdot Z^{4-5} \cdot \varepsilon_\gamma^{-7/2}$$

$$\varepsilon_\gamma = \frac{h\nu}{m_e c^2}$$

1. photon disappears

   photo electron $E_e = E_{photon} - E_B$

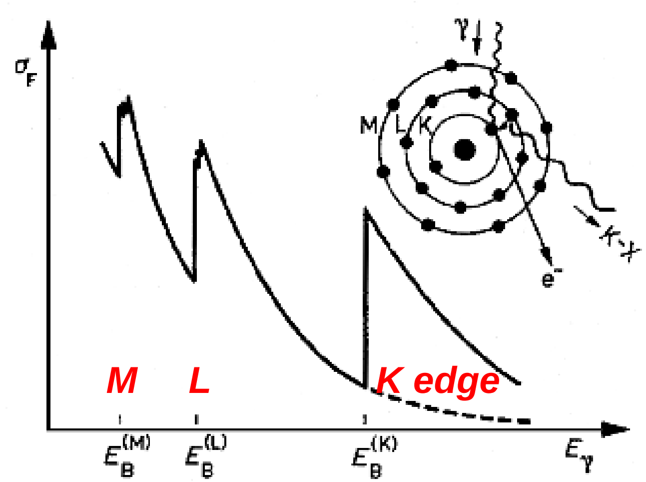
2. refilling of hole in electron shell by

   a) emission of photon or

   b) Auger electron emission of

   loosely bound outer electron

   $E_{Auger} \cong E_B$

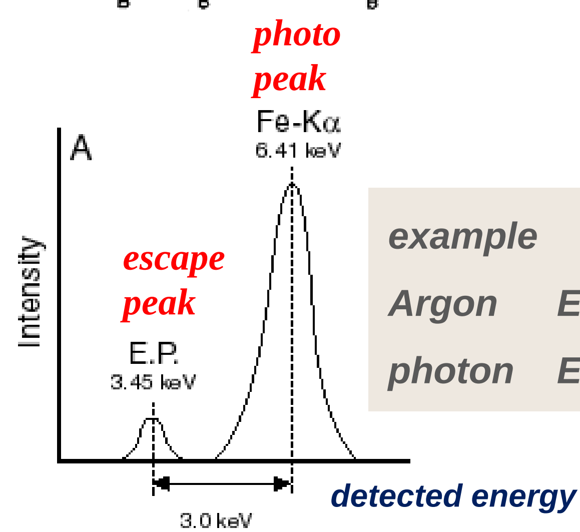


detected energy E

photo peak $E = E_{photon}$

$= E_e + E_B$

escape peak $E = E_{photon} - E_{K\alpha}$

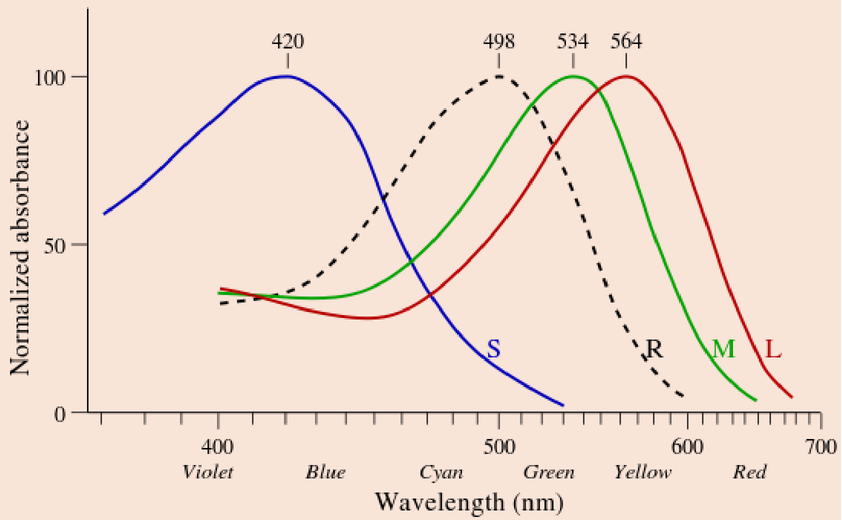


example

Argon	$E_{K\alpha} = 2.95$ keV
photon	$E_{Photon} = 6.41$ keV

RESPONSE of EYE CELLS

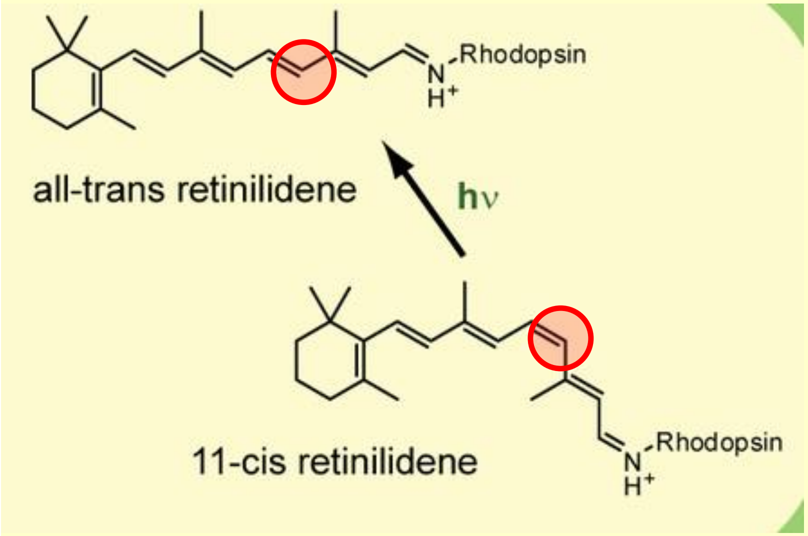
sensitivity 1 photon



cone cells

rod cells

sensitivity 100 photons



cis – trans transition

of various opsin (protein) configurations

COMPTON EFFECT

proof of particle nature of „light“ *Compton 1922*

billard with photons and „quasifree“ electrons

$$\sigma_C \approx \sigma_{Th} \cdot (1 - 2\varepsilon\gamma + \cdots) \cdot Z \qquad \varepsilon_\gamma \ll 1$$

$$\approx \sigma_{Th} \cdot \frac{3}{4} \cdot \left(\frac{1+2ln\varepsilon\gamma}{2\varepsilon\gamma} + \cdots\right) \cdot Z \qquad \varepsilon_\gamma \gg 1$$

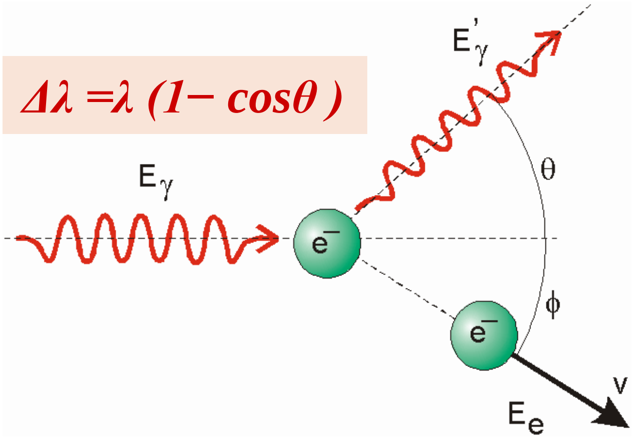
complicated QED calculation Klein&Nishina 1929

photon does not disappear

recoil electron $\quad E_e = E_{photon} - E_{photon'}$

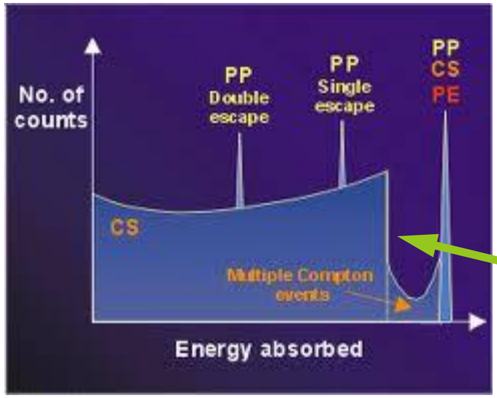
$\Rightarrow$ *continuous spectrum*

$$\Delta\lambda = \lambda\,(1 - \cos\theta)$$



detected energy $\quad E = E_e$

we neglegt E_B of the electron and E_{recoil} of the nucleus because usually E_B, E_{recoil} << E_e



Compton edge
= maximum energy transfer

PAIR PRODUCTION

proof of mass-energy equivalence ***Blackett 1948***

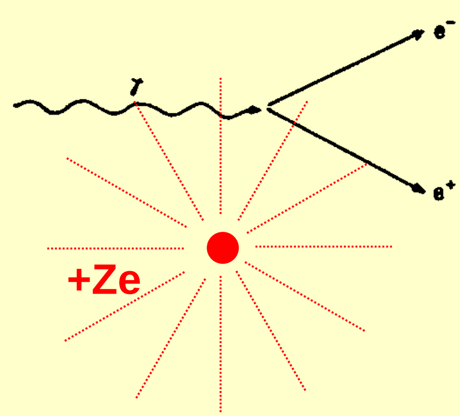
$$\sigma_{pair} \approx \sigma_{Th} \cdot Z^2 \cdot (\ln 2\varepsilon\gamma + \cdots) \quad \varepsilon_\gamma \gg 1$$

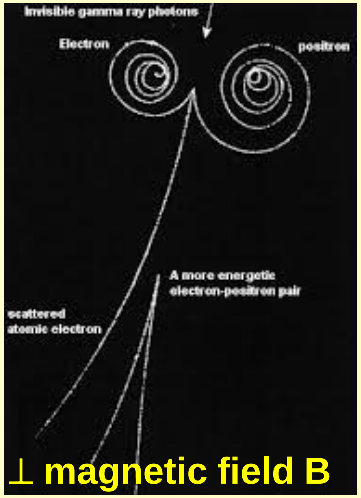
conversion of energy into matter

$E_{photon} = h\nu > 2\ m_{electron,muon,pion,\ \ldots}$

a recoil partner (e.g. a nucleus) is needed

to fulfil energy and momentum conservation





e^+e^- threshold: $m_{recoil} = \infty \quad h\nu = 2\ m_ec^2$

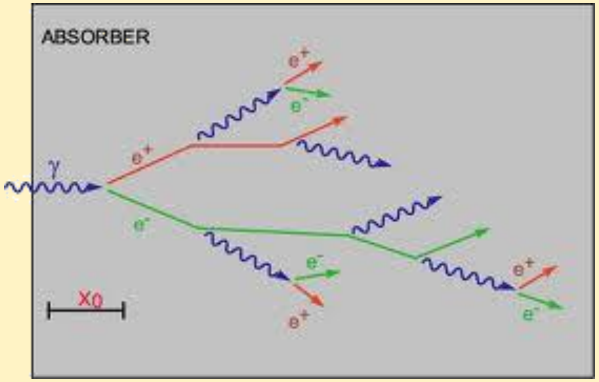
$\phantom{e^+e^- threshold: m_{recoil}} = m_e \quad\quad = 4\ m_ec^2$

el.-mag. shower

$e^+\ e^-\ \gamma$ - cascade

pair production and bremsstrahlung alternate

shower may start with photon or electron

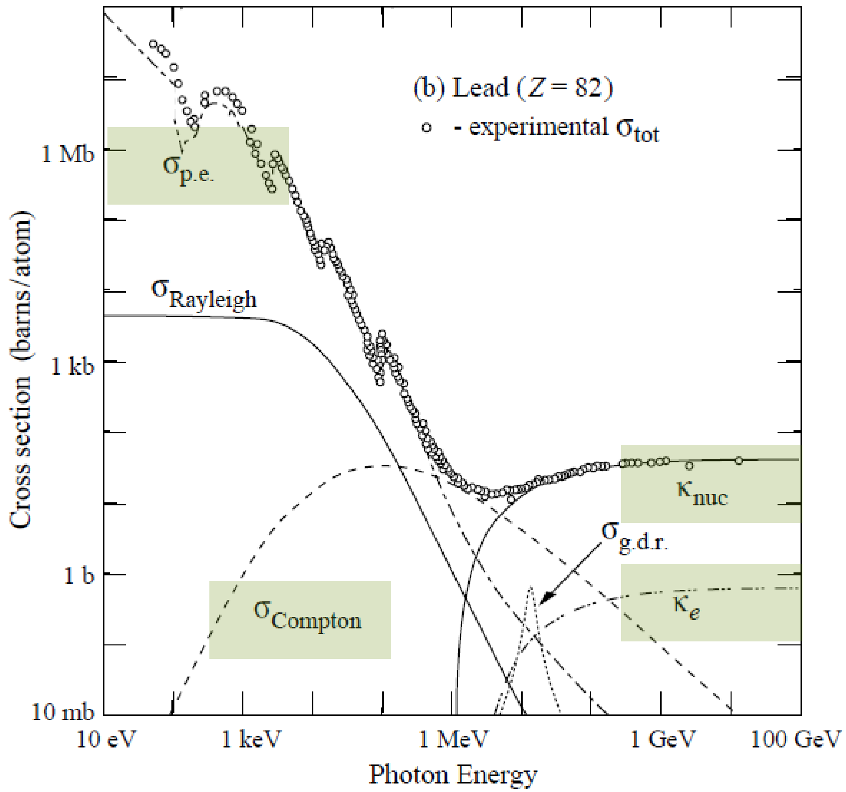


radiation length X_0

characteristic material dependent constant

depth, where about 2/3 of the incident energy is converted

CROSS SECTIONS SUMMARY



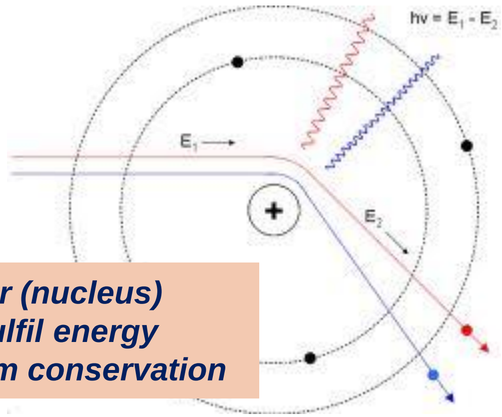
$$
\begin{aligned}
\sigma_{\text{p.e.}} &= \text{Atomic photoelectric effect (electron ejection, photon absorption)} \\
\sigma_{\text{Rayleigh}} &= \text{Rayleigh (coherent) scattering–atom neither ionized nor excited} \\
\sigma_{\text{Compton}} &= \text{Incoherent scattering (Compton scattering off an electron)} \\
\kappa_{\text{nuc}} &= \text{Pair production, nuclear field} \\
\kappa_{e} &= \text{Pair production, electron field} \\
\sigma_{\text{g.d.r.}} &= \text{Photonuclear interactions,}
\end{aligned}
$$

BREMSSTRAHLUNG

accelerated charged particles radiate *Hertz 1886*

$\sigma_b \approx \sigma_{Th} \cdot Z^2 \cdot [energy\ dependent]$

electromagnetic waves

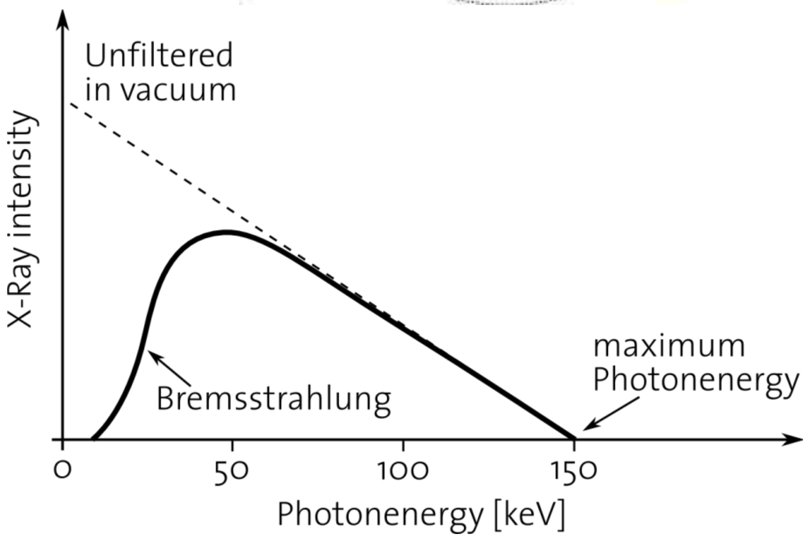


bending force by Coulomb potential

force ⇔ acceleration

a recoil partner (nucleus) is needed to fulfil energy and momentum conservation

$$F_{Coulomb} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_{particle} \cdot Q_{nucleus}}{r^2} = m \cdot \ddot{r}$$



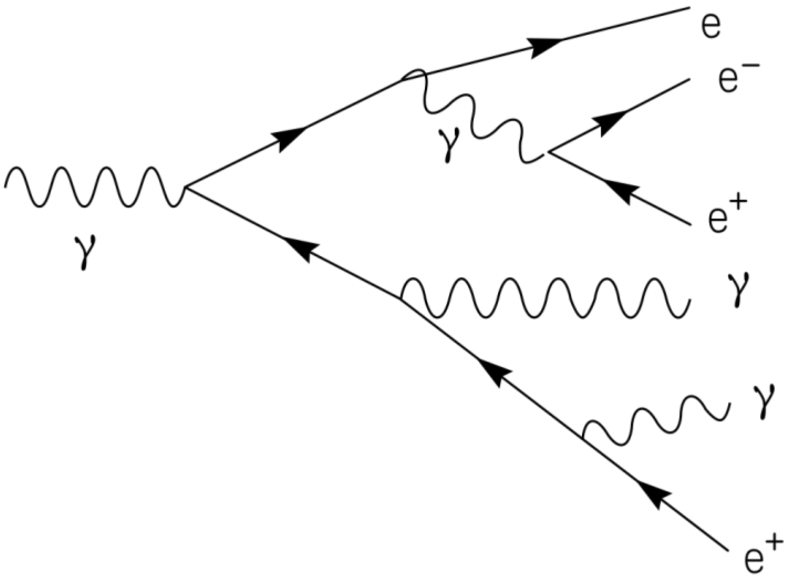
any distance r

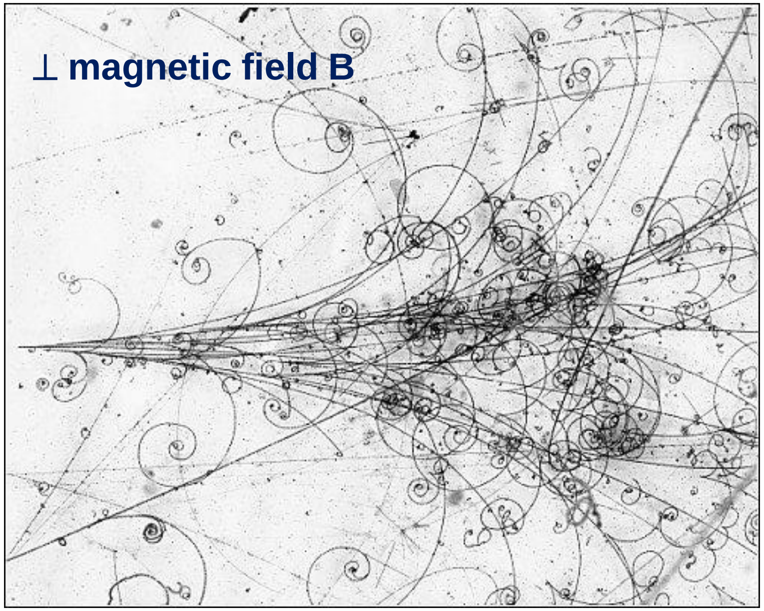
⇒ continuous spectrum

EL.-MAG. SHOWER

alternating pair production & bremsstrahlung

initial particle of minor importance for large energies





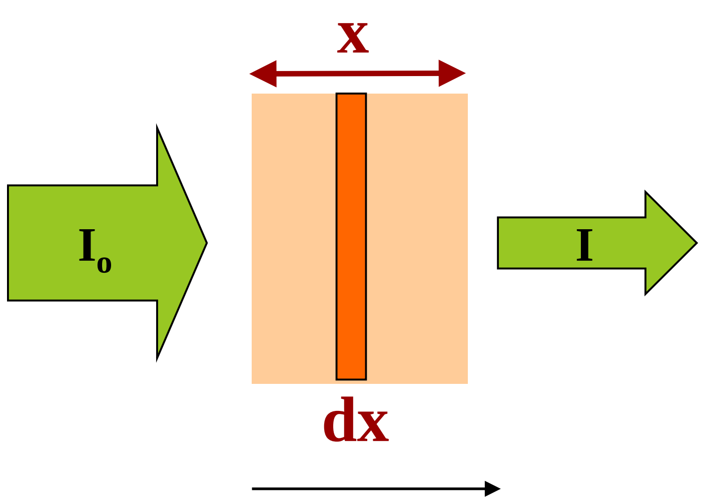
characteristic quantity of absorber

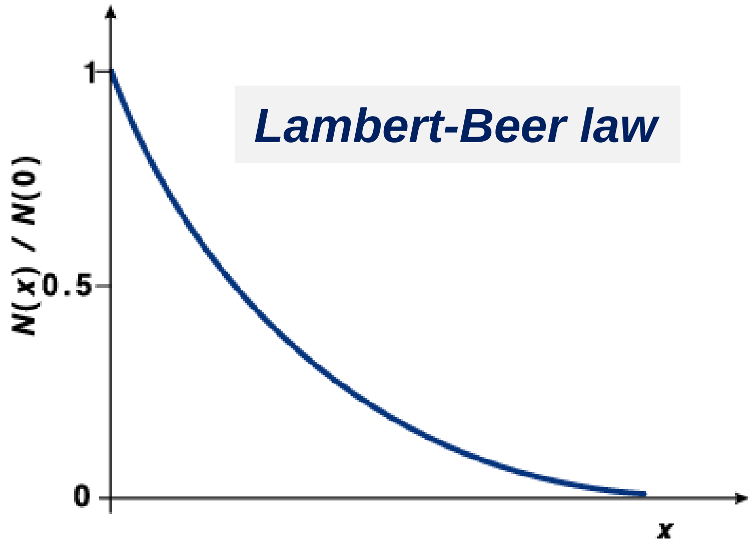
radiation length X_0

x

$$E_\gamma = E_{initial} \cdot e^{-(x/X_0)}$$

ATTENUATION





$$\mathrm{I(x)} = \mathrm{I_0 e^{-\mu(h\nu)x}}$$ *intensity after layer thickness x*

$$\frac{\mathrm{I_T(x)}}{\mathrm{I_0}} = \mathrm{e^{-\mu(h\nu)x}}$$ *fraction of transmission*

$$\frac{\mathrm{I_A(x)}}{\mathrm{I_0}} = 1 - \mathrm{e^{-\mu(h\nu)x}}$$ *fraction of absorption*

sum of linear attenuation coeff.

$$\mu(h\nu) = \sum_i \mu_i(h\nu)$$

$$\mu_i = \rho \cdot \frac{N_A}{A} \cdot \sigma_i(h\nu)$$

INTERACTION of MASSIVE PARTICLES

CHARGED PARTICLES : ENERGY LOSS BY IONIZATION

HEAVY CHARGED PARTICLES

LIGHT CHARGED PARTICLES

CHARGED PARTICLES : ENERGY LOSS BY RADIATION

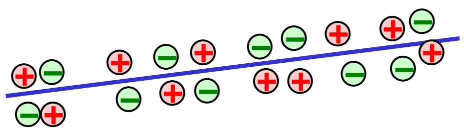
NEUTRONS

CHARGED PARTICLES - ENERGY LOSS by IONIZATION

collisions create electron- ion pairs

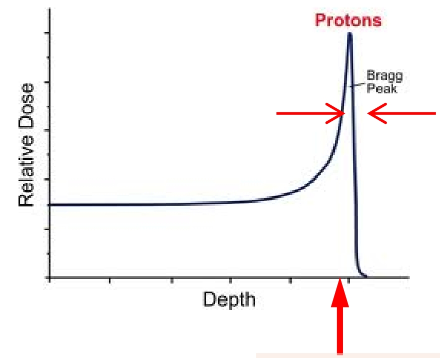
1. ***heavy*** **$M_{particle} >> M_{electron}$**

e.g. protons, deuterons, …



strongly ionising

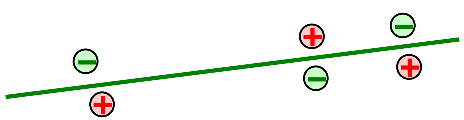
Bragg peak



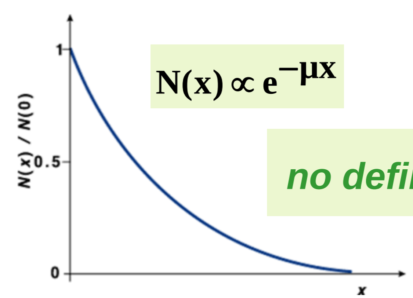
well defined range R!

2. ***light*** **$M_{particle} = M_{electron}$**

electrons or positrons



weakly ionising



$$N(x) \propto e^{-\mu x}$$

no defined range R!

exponential attenuation with depth x

μ: material dependent attenuation coefficient

HEAVY CHARGED PARTICLES - STOPPING POWER I

heavy particles ***μ, π, K, p, d, …***

stopping power

$$S = \left(-\frac{dE}{dx}\right)\cdot\frac{1}{\rho}$$

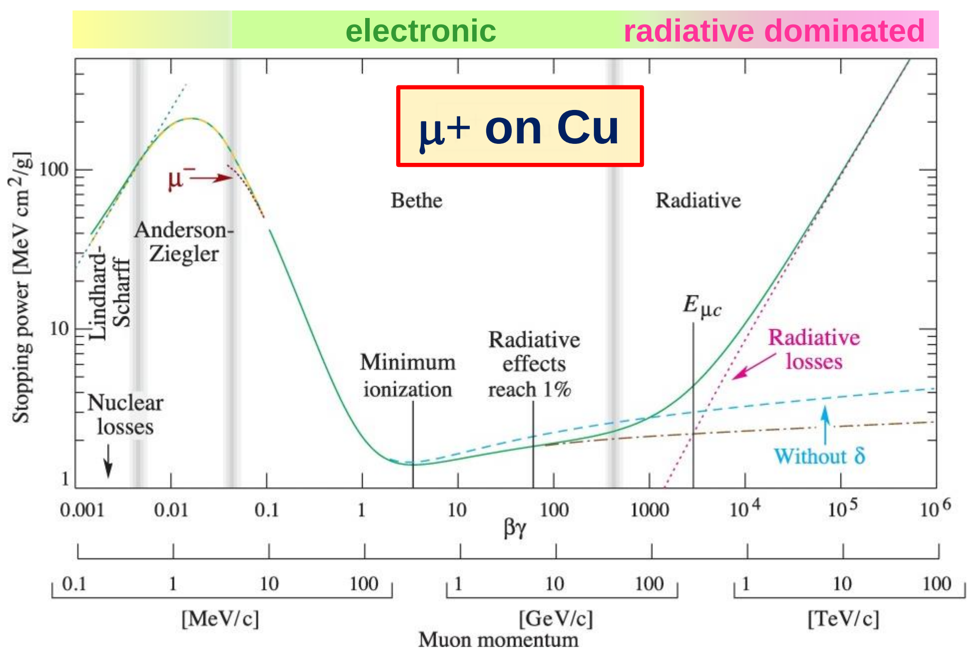
[MeV· cm²/g]

$\beta < \alpha \approx \beta_{\text{valence electron}}$

$\alpha < \beta < 0.1$ β_{electron} **neglected**

$\beta = 0.7$

$\beta = 1 - 10^{-7}$



HEAVY CHARGED PARTICLES - STOPPING POWER II

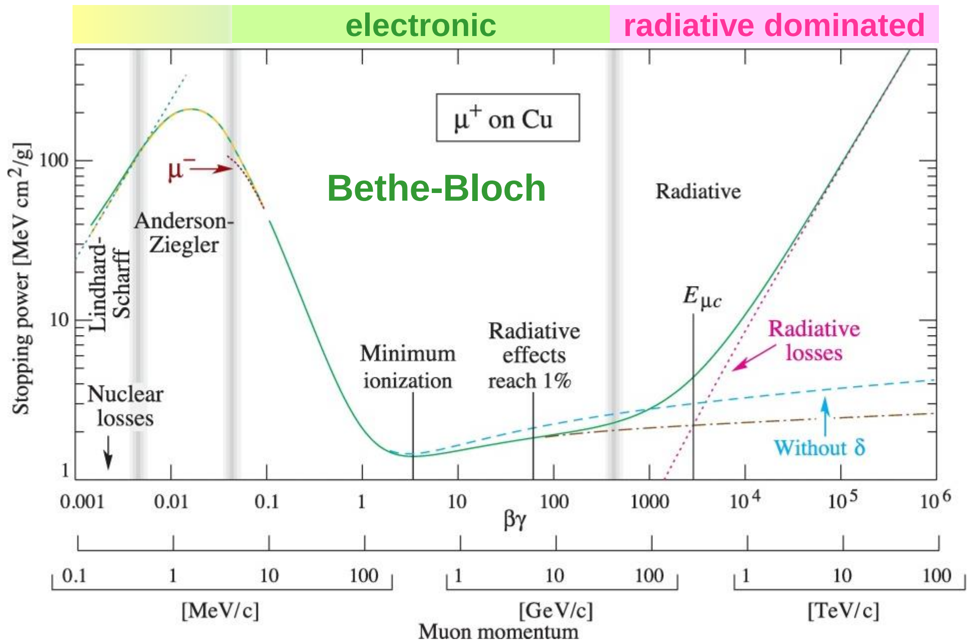
Bethe-Bloch range

$\alpha < \beta < 0.1$: $\beta_{electron}$ neglected

stopping power

$$S = \left(-\frac{dE}{dx}\right) \cdot \frac{1}{\rho}$$

$[\mathrm{MeV} \cdot \mathrm{cm}^2/\mathrm{g}]$



C_0 shell corrections for low energies

$$S = 4\pi N_A r_e^{\,2} m_e c^2 \frac{Z_{target}}{A} \cdot \frac{z^2{}_{projectile}}{\beta^2} \cdot \left[\frac{1}{2} ln\left(\frac{2m_e c^2 \beta^2 \gamma^2 T_e^{\,max}}{I^2}\right) - \beta^2 - \frac{C_0(\beta^2)}{Z_{target}} - \frac{\delta(\beta\gamma)}{2}\right]$$

HEAVY CHARGED PARTICLES - STOPPING POWER III

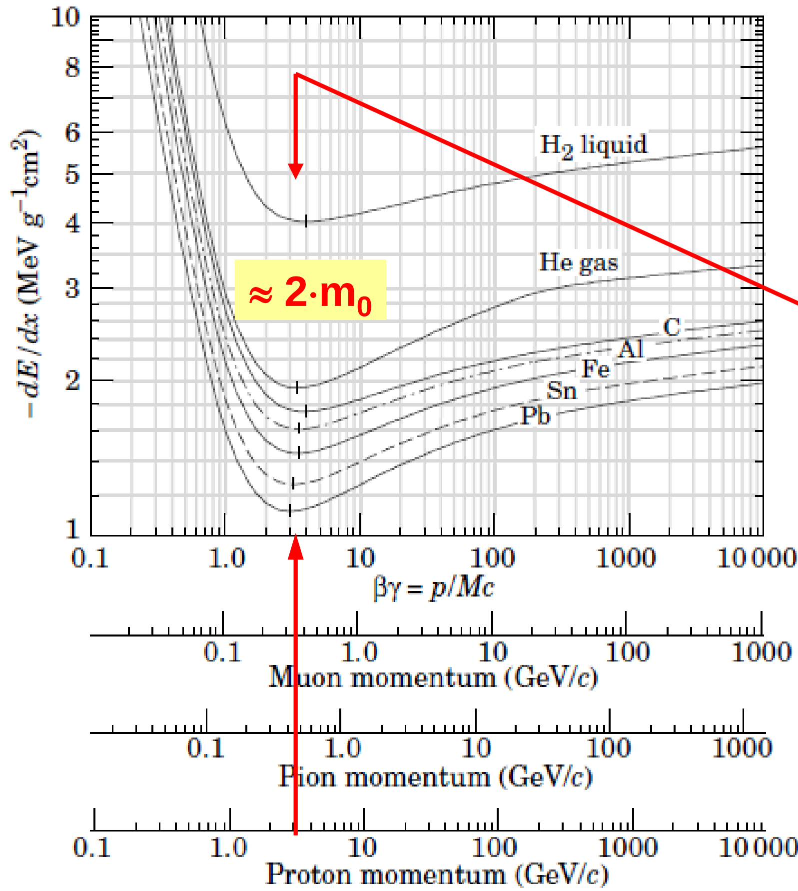


Figure 30.2: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 30.23.

Bethe-Bloch range

$$\left(\frac{\Delta E}{\Delta x}\right)_{\text{collision}} \propto \frac{1}{v^2} \quad \ldots$$

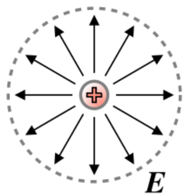
MIPs = *minimum ionising particles*

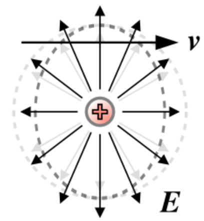
$$\left(-\frac{dE}{dx}\right)_{min} = 4\ MeV g^{-1} cm^2 \qquad \textit{for}\ \ A = 1$$

$$\left(-\frac{dE}{dx}\right)_{min} = 1 - 2\ MeV g^{-1} cm^2 \quad \textit{for}\ \ A > 1$$

at $\beta\gamma = 3 - 3.5$ *or* $T_{min} = 2.2 - 2.6\ m_0c^2$

HEAVY CHARGED PARTICLES - STOPPING POWER IV





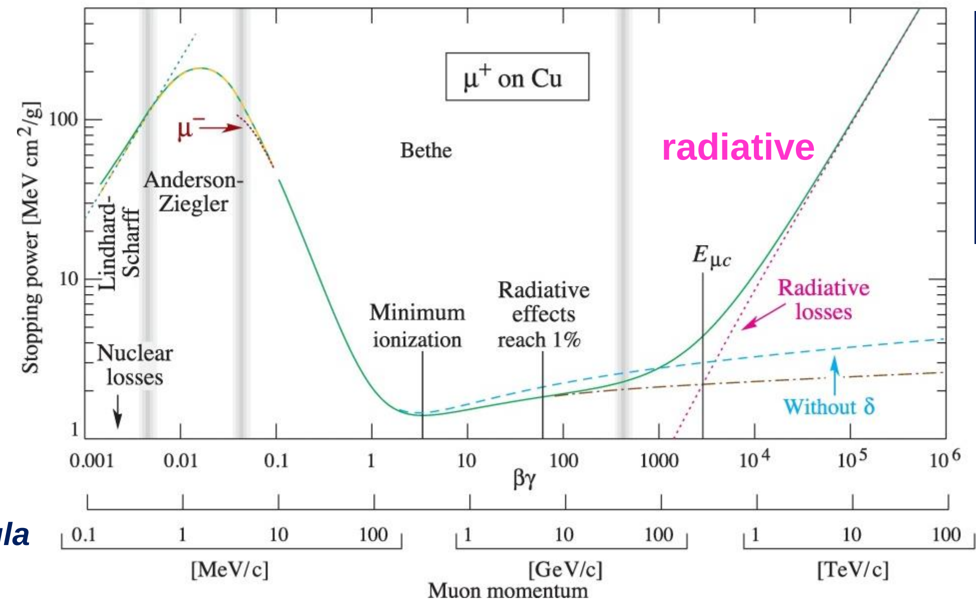
$$E_\perp = \gamma E_0$$
$$E_\parallel = E_0$$

„LHC" range

radiative losses

- *bremsstrahlung*
- *pair production e^+e^-*
- *photonuclear*

<u>not</u> covered by this formula

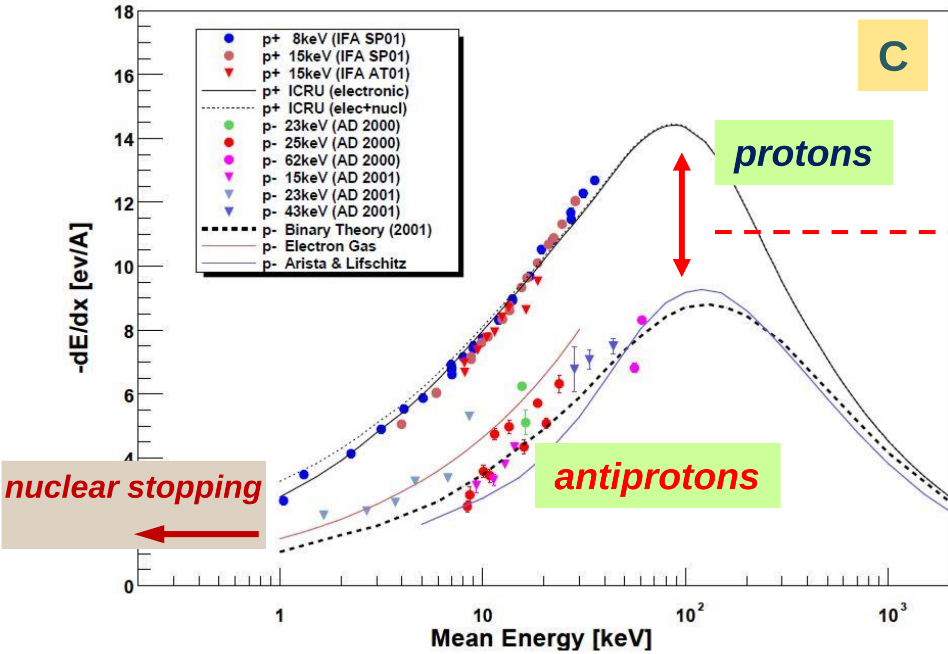


δ density effect

polarization diminishes relativistic rise

$$S = 4\pi N_A {r_e}^2 m_e c^2 \frac{Z_{target}}{A} \cdot \frac{z^2{}_{projectile}}{\beta^2} \cdot \left[\frac{1}{2} ln \left(\frac{2 m_e c^2 \beta^2 \gamma^2 {T_e}^{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

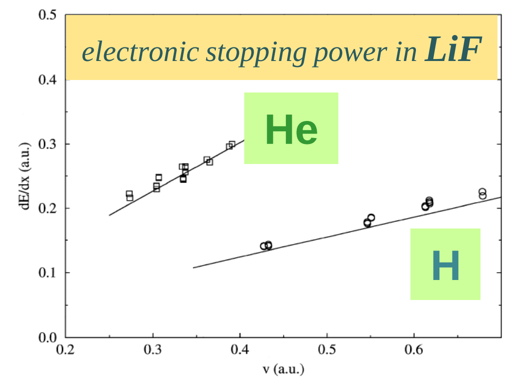
HEAVY CHARGED PARTICLES - BARKAS EFFECT



A.Csete / PhD thesis, Aarhus, 2002

lowest energies - friction range

like friction
$$-\left(\frac{\Delta E}{\Delta x}\right) \propto \mathbf{v}$$



Juaristi et al, PRL 84, 2124 (2000)

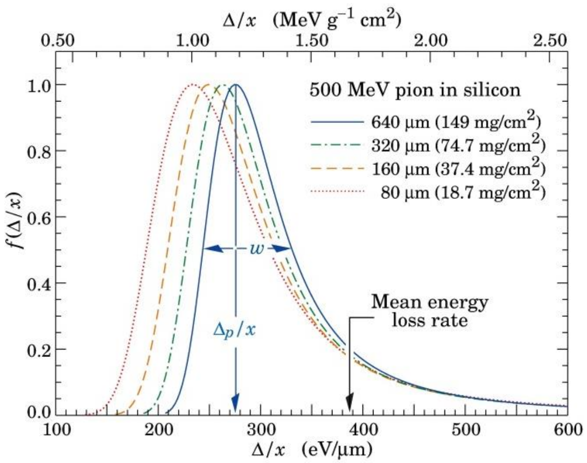
sensitive to sign of charge

$$S = 4\pi N_A r_e^2 m_e c^2 \frac{Z_{target}}{A} \cdot \frac{z^2_{projectile}}{\beta^2} \cdot \{[\ldots] + L_1(\beta, Z_{target}) \cdot z_{projectile}\}$$

frictional cooling (e-cooler, muon collider), window design, …

HEAVY CHARGED PARTICLES - STRAGGLING

energy straggling



Landau-Vavilov distribution

asymmetric energy straggling towards higher ΔE

thick layers → many collisions → skewness decreases

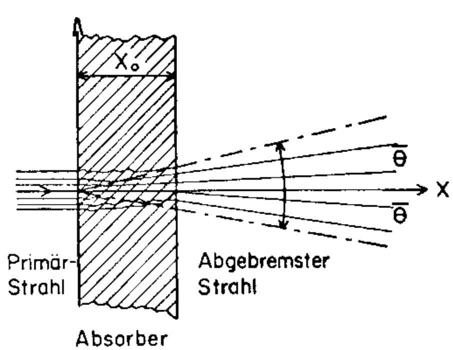
Δ_p/x *most probable energy loss (here normalized to unity)*

Δ/x *energy loss per layer thickness*

$$\overline{\Delta}^2 \propto \frac{Z}{A}\cdot\rho\cdot d\cdot\frac{1}{\beta^2}$$

for „thin" layers

angular straggling



$$\overline{\Theta} = \frac{13.6\ MeV}{\beta cp}\sqrt{x/X_0}\,(1+\ldots) \propto z\cdot Z/p^2$$

many collisions → Gaussian angular distribution

X_0 / gcm^{-2}	= 63 (126)	H_2 (D_2)	radiation length
	= 108	Si	
	= 13.8	Fe	

x / gcm^{-2} effective thickness of layer ($x = d\cdot\rho$)

acceptance of experimental setup (storage rings etc.)
position resolution of tracking devices

HEAVY CHARGED PARTICLES - RANGE I

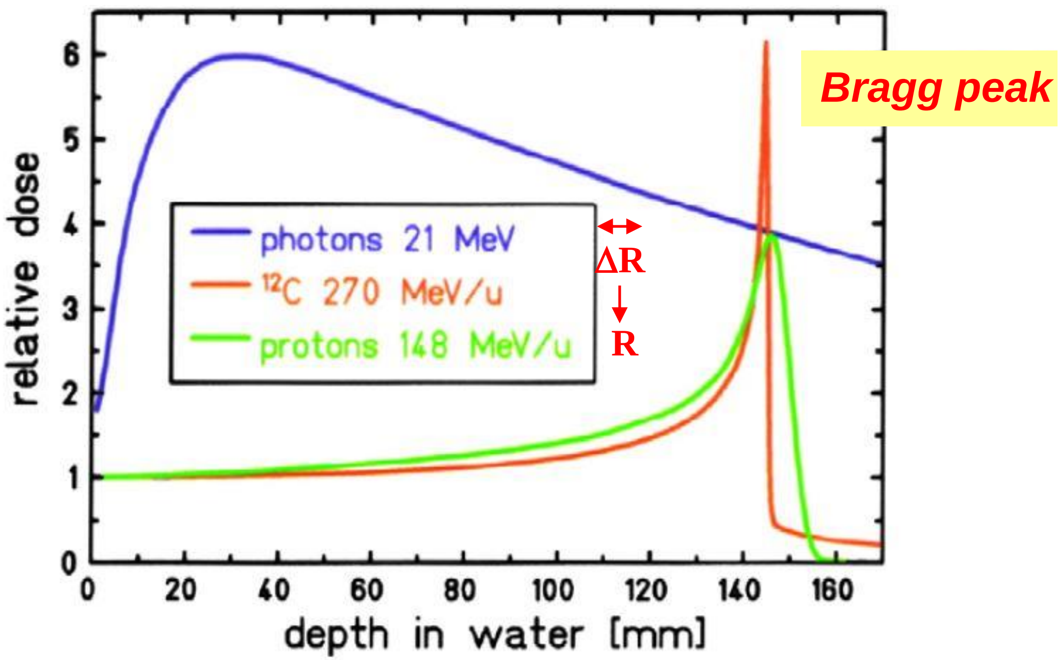


Fig. 1. Depth dose distribution for photons and monoenergetic Bragg curves for carbon ions and protons (Courtesy of G. Kraft, GSI Darmstadt, Germany).

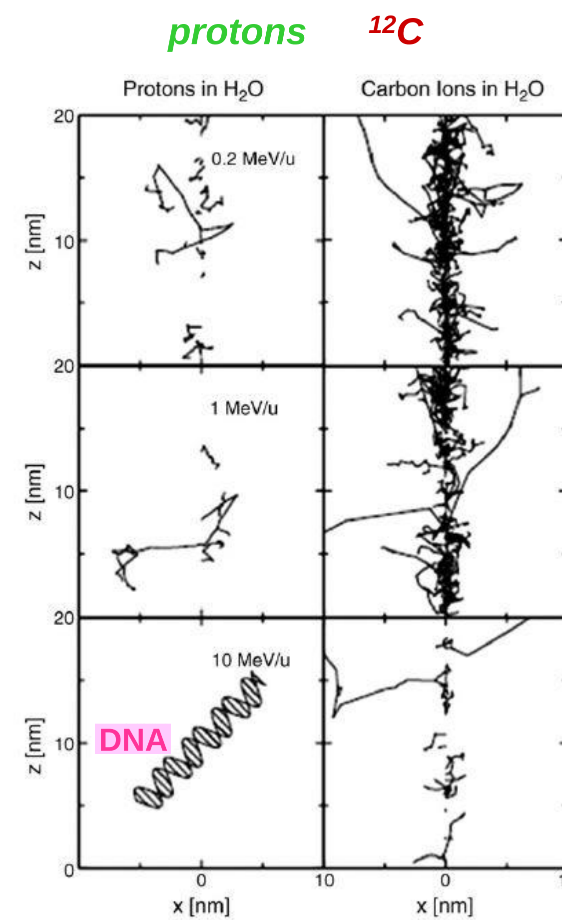


Fig. 4. Proton and carbon ion tracks are compared microscopically to an illustration of a DNA molecule before, in and behind the Bragg maximum, for the same energy [41].

Biochimica et Biophysica Acta 1796 (2009) 216–229

HEAVY CHARGED PARTICLES - RANGE II

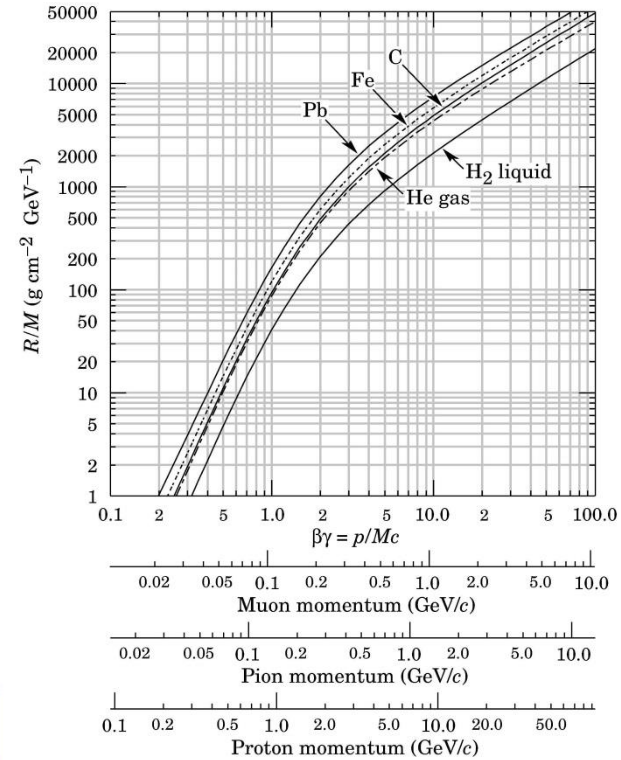
mean range $\qquad R = \int_{T_{kin}}^{0} dE / (dE/dx)$ [cm]

depends on particle mass

range – straggling $\qquad \Delta R$

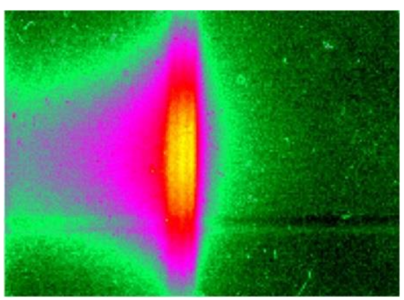
longitudinal $\qquad \Delta R/R \approx$ *1% – 3%* $\qquad$ ***for all elements***

transversal $\qquad \approx$ *2% – 6%*



47 MeV antiprotons

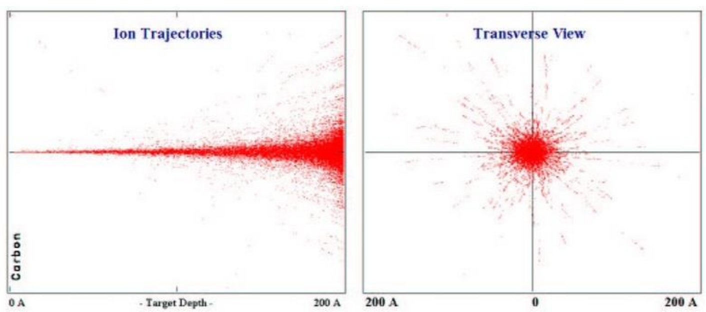
radiochromic film response



N. Bassler et al.
Radiotherapy and Oncology 86 (2008) 14–19

20 keV protons on carbon

(Monte-Carlo simulation SRIM)



A.Csete / PhD thesis, Aarhus, 2002

R/M(E/M)

range concept useful for

- R < λ_{had}

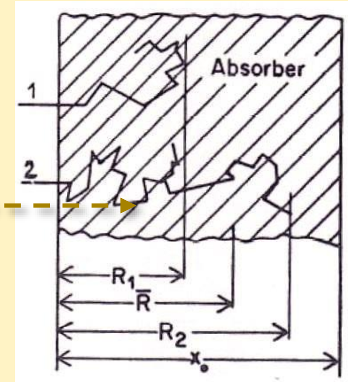
- radiation losses small

LIGHT CHARGED PARTICLES - STOPPING POWER

<u>ionisation</u> dominated energy range

heavy particles path lengths **S ≈ range R**

electrons/positrons " " **S ≈ 2 · R**



strong deflection

dE/dx similar to Bethe-Bloch formula

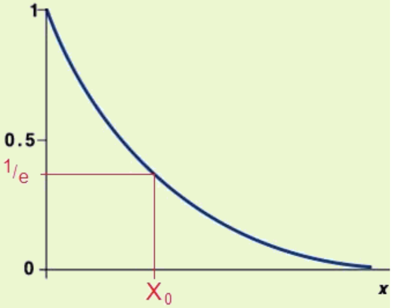
additional terms ***- identical particles (e^-)***
- spin dependence

<u>radiation</u> dominated energy range

energy loss by bremsstrahlung $$-\frac{dE_{kin}}{dx} \propto Z^2{}_{target} \cdot E_{kin} \cdot [\ldots]$$

$$\Rightarrow \quad E_{kin} = E_{0,\,kin} \cdot e^{-\left(x/X_0\right)}$$

radiation length X_0 [g·cm^{-2}] $$\frac{1}{X_0} = 4\alpha \cdot r_e{}^2 \cdot \frac{N_A}{A} \cdot Z^2{}_{target} \cdot [\ldots]$$



after depth d = X_0/ρ ([cm]) all but 1/e of the energy of the particle is lost by bremsstrahlung

LIGHT CHARGED PARTICLES - RANGE

ionisation dominated energy range

electron range ***(semiempirical formulae)***

$$R = 0{,}52\, E^{(\mathrm{MeV})} - 0{,}09 \quad (\mathrm{g\,cm^{-2}}) \qquad 0{,}5 < E_e < 3\ \mathrm{MeV}$$

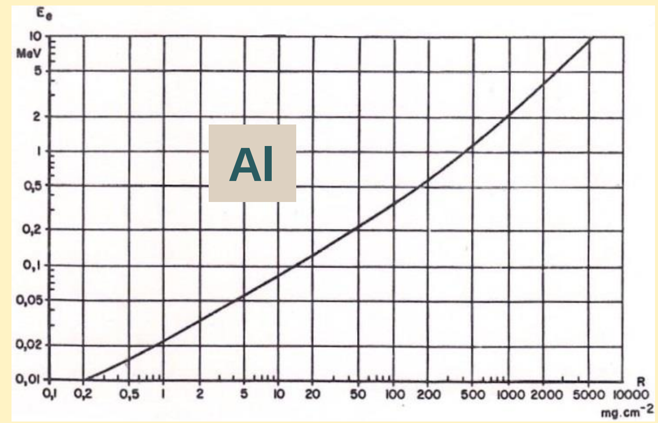
$$R = 0{,}412\, E^{n} \quad (\mathrm{g\,cm^{-2}}) \qquad 0{,}01 < E_e < 3\ \mathrm{MeV}$$

$$\text{mit} \quad n = 1{,}265 - 0{,}0954 \ln E_e$$

$$R = 0{,}53\, E^{(\mathrm{MeV})} - 0{,}106 \quad (\mathrm{g\,cm^{-2}}) \qquad 1 < E_e < 20\ \mathrm{MeV}$$

$$-\frac{dE}{dx} = \frac{2\pi e^4}{E_e} N^0 Z \left(\ln \frac{E_e}{I} + 0{,}15 \right) \qquad E_e \ll m_e c^2$$

$$-\frac{dE}{dx} = \frac{2\pi e^4}{m_e c^2} N^0 Z \left(\ln \frac{E_e^3}{2 m_e c^2 I^2} + \frac{1}{8} \right) \qquad E_e \gg m_e c^2$$

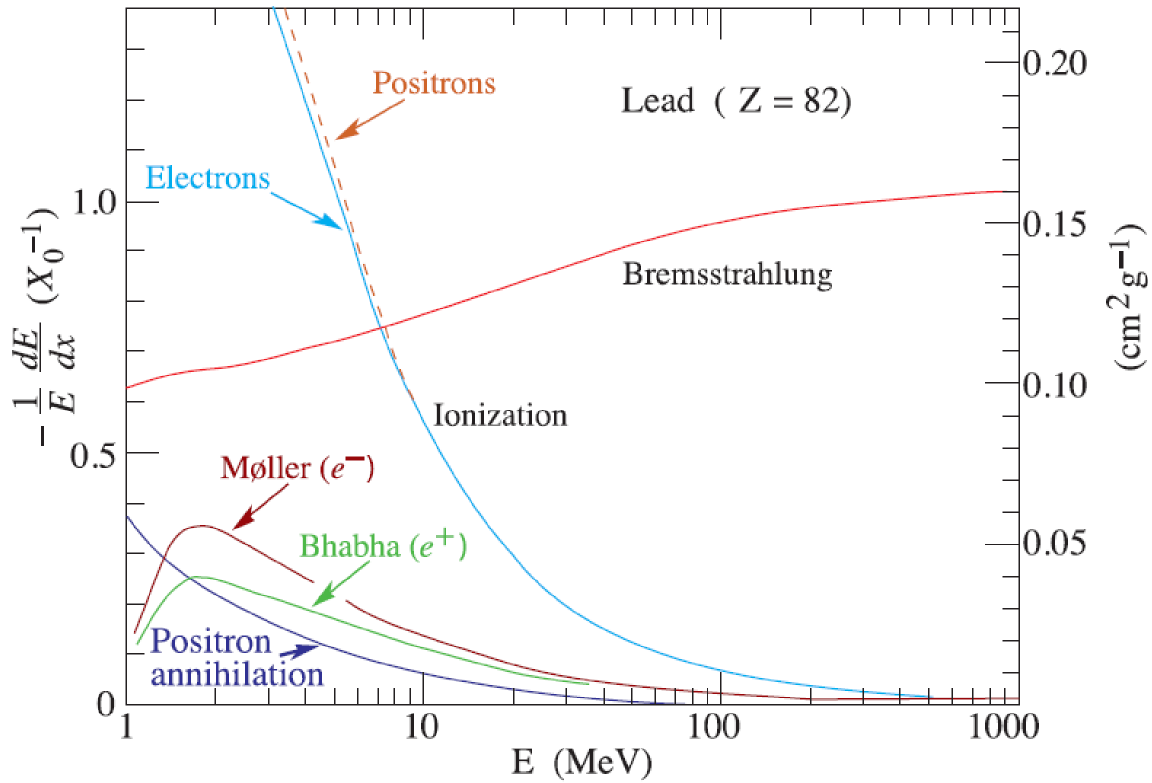


radiation dominated energy range

radiation length X_0 [g·cm^{-2}]

D_2	126	mylar	40
H_2	63	air	37
Al	24	water	36
Ar	20	rock standard	27
Cu	13	CsI	8.4
Pb	6	$PbWO_4$	7.4

LIGHT CHARGED PARTICLES - RELATIVE ENERGY LOSS



Fractional energy loss per radiation length in lead as a function of electron or positron energy.

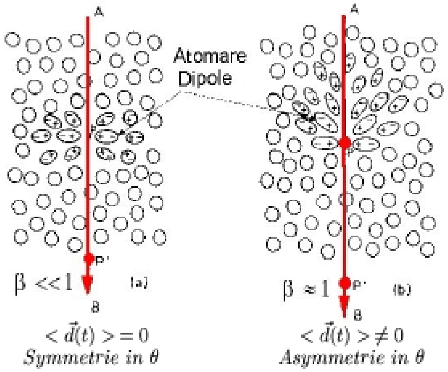
CHARGED PARTICLES - ENERGY LOSS BY RADIATION I

Cerenkov radiation if $v_{particle} > c_{in\ medium}$

Cerenkov 1930s

„light" blue!

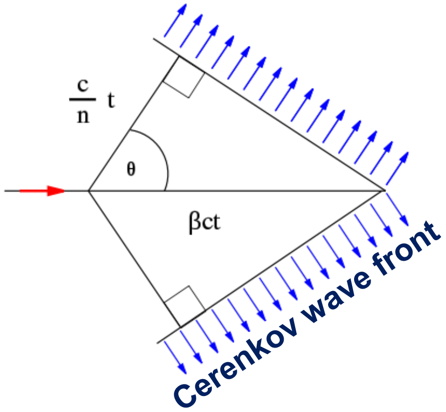
the charge polarizes the medium



electrons „radiate"
in the water above
the core of
a nuclear power plant

$$\left(\frac{\Delta E}{\Delta x}\right)_{C\,radiation} << \left(\frac{\Delta E}{\Delta x}\right)_{collision}$$

emission under specific angle Θ_C



$\cos \Theta_C = 1 / \beta \cdot n$

n = index of refraction

(small) dispersion !

Θ_C measures the velocity of the particle

acoustics analogue: Mach's cone for supersonic source

CHARGED PARTICLES - ENERGY LOSS BY RADIATION II

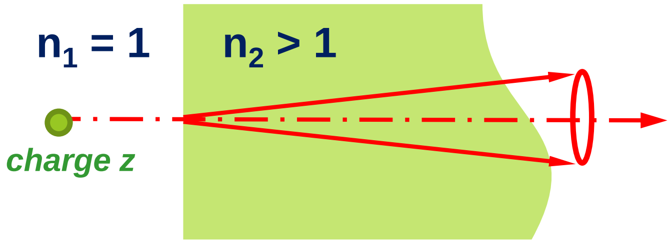
Transition radiation for ultrarelativistic particles ($\gamma \gg 1$)

Ginzburg & Frank 1946

Readjustment of the el.-mag. fields (E,H) at the boundary of 2 media

with different dielectric properties (ε)

leads as collective response of the material to *emission of el.-mag. radiation (X-rays)*



emission cone $\Theta \approx 1/\gamma \approx 10^{-3}$ rad

radiated intensity $I = \alpha \cdot z^2 \cdot \gamma \cdot \hbar\omega_P /3$

photon yield $n_{Photon} \propto \alpha \cdot z^2 \cdot (\ln \gamma)^2 \approx z^2 \cdot 0.5\%$

plasma frequency $\omega_P^2 = \frac{e^2}{\varepsilon_0} \cdot \frac{n_e}{m_e}$ $air: \quad \hbar\omega_P = 0.7\ eV$

$mylar: \hbar\omega_P = 20\ eV$

formation length $d = \frac{\gamma}{\sqrt{2}} \cdot \frac{\hbar c}{\hbar\omega_P}$ $mylar: d = 14\ \mu m\ (\gamma = 1000)$

typical: *soft X-rays of 2-40 keV for $\gamma \approx 1000$*

application: plasma frequencies of materials, particle separation (π/p), …

NEUTRONS I

collisions create recoil particles

maximum energy transfer for $M_{neutral} = M_{recoil}$

central collision	*energy is transferred completely*
non central	*all energies according to scattering angle*
averge energy transfer	*50%*

detection by recoil protons (from hydrogen)

$$M_{Proton} \approx M_{Neutron}$$

i.e. ***good shieldings are water***

concrete (15% water)

paraffin ($(CH)_n$)

…

cloud chamber picture

neutron

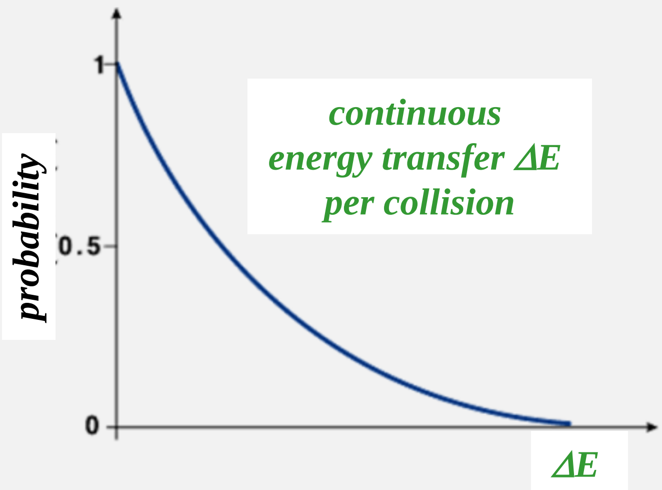
NEUTRONS II

slowing down of neutrons in elastic collisions

$$\left(\frac{A-1}{A+1}\right)^2 \cdot Tn \le Tn' \le Tn$$

T_n initial kinetic energy

T_n' kinetic energy after collision



neutrons – no defined range

make degrader thick enough to themalize neutrons, i.e., $T_n \approx \frac{1}{40}\ eV$

subsequent capture or decay

don't forget absorber for reaction and decay products (mostly γ)

APPENDIX

UNITS

	atoms		nuclei or particles
length	1 nm = 10^{-9} m		1 fm = 10^{-15} m (= 1 Fm)
- *Bohr radius*	*$\approx 0.5\ 10^{-10}$ m = 0.05 nm*		
- *proton „radius“*			*$\approx 0.8\ 10^{-15}$ m = 0.8 fm*
energy	eV		MeV
- *binding energy*	*- 13.6 eV $\cdot Z^2 / n^2$*		*$\approx$ 8 MeV / nucleon.*
- *visible light*	*1 – 2 eV*		*γ – rays up a few MeV*
- *X-rays*	*up to $\approx$ 100 keV*		
momentum	eV/c		MeV/c
time	1 fs = 10^{-15} s		1 ys = 10^{-24} s
- *life time*	*2p hole in K atom $\approx$ 6 fs*		*Δ baryon $6 \cdot 10^{-24}$ s*
mass	a.m.u. $\equiv$ m(^{12}C)/12		MeV/c^2
rest mass m_0	*$\approx A \cdot$ a.m.u.*	*- a.m.u.*	*931.5* MeV/c^2 *= $1.66 \cdot 10^{-27}$ kg*
		- proton	*938.3* MeV/c^2 *$\approx$ 1 a.m.u.*
		- neutron	*939.6* MeV/c^2 *$\approx$ 1 a.m.u*
		- electron	*0.511* MeV/c^2 *= $9 \cdot 10^{-31}$kg $\approx$ 1/1823 a.m.u*
angular momentum (Spin)	j = 0,1/2,1,3/2, ... $\hbar$		**[j]=[E·t]**
magnetic moment	$\mu_B = 5.8 \cdot 10^{-11}$ MeV·T^{-1}		$\mu_N = 3.2 \cdot 10^{-14}$ MeV·T^{-1}
cross section	$\boldsymbol{\sigma}$		*10^{-24} cm^2 = 1 barn (b)*
			1 a. u. = r_B^2

CONSTANTS

speed of light ***definition!***	$c \equiv 299\,792\,458$ m/s	≈ 0.3 m/ns
elementary charge ($\pm$)	$e = 1.6 \cdot 10^{-19}$ C	
Avogadro constant	$N_A = 6.02 \cdot 10^{23}$ / mol	
Planck constant	$h = 6.6 \cdot 10^{-34}$ J·s	
	$\hbar = h/2\pi$	analogue $\lambdabar$, …
	$\hbar = 6.582 \cdot 10^{-22}$ MeV·s	
fine-structure constant	$\alpha = e^2/4\pi\varepsilon_o\hbar c$	$= 1/137$
classical electron radius	$r_e = e^2/4\pi\varepsilon_o m_e c^2 = \alpha\,\hbar c/\,m_e c^2$	$= 2.82$ fm
conversion constant	$\hbar c = 197$ MeV·fm ($\hbar = h/2\pi$)	
conversion constant	$hc = 12.4$ keV·10^{-10} m	

RELATIONS

wave propagation	c = $\lambda\nu$	
Photon energy	E = $h\nu = \hbar\,\omega$	
uncertainty relation	$\Delta p \cdot \Delta x \geq \hbar$	$\Delta E \cdot \Delta t \geq \hbar$
life time $t \Leftrightarrow \Delta E \equiv \Gamma$	$\Gamma \cdot \tau = \hbar/2$	MeV·s
Bohr formula	$E_B = -mc^2 \cdot \frac{(Z\alpha)^2}{2n^2}$	$Ryd = m_e c^2 \cdot \frac{\alpha^2}{2} = 13.6\ eV$
Bohr radius	$r_B = \frac{\hbar c}{\alpha m_e c^2} mc^2 = r_e \alpha^{-2}$	$= 0.53 \cdot 10^{-10}\ m$
Virial theorem	$2 \cdot \langle T_{kin} \rangle = n \cdot \langle V_{tot} \rangle$	$\langle \ldots \rangle$ *average of* ...
count rate	$N = N_{in} \cdot \rho d \frac{N_A}{A} \cdot \sigma$	

RELATIVISTIC KINEMATICS

	massive particles		*el.-mag. radiation*	
total energy	$E_{total} = \sqrt{p^2c^2 + m_0^2c^4}$ $= \gamma m_0 c^2$		$E_{total} = pc$ $= h\nu$ $= E_{kin}$	
kinetic energy	$E_{kin} = E_{total} - m_0 c^2$			
momentum	$p = \gamma m_0 c \cdot \beta$		$p = \frac{E}{c}$	
rest mass	$m_0 \neq 0$	*range in matter*	$= 0$	*attenuation in matter*
charge	$Q \neq 0$	*deflection in el.-mag. fields*	$= 0$	*no deflection*
life time	$\tau = \gamma \tau_0$	*decay length* $l = v\tau$	$= \infty$	

relativistic factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$ $\lim \gamma \xrightarrow[v \to c]{} \infty$

LITERATURE

G. F. Knoll — **Radiation detection and measurement**

4th edition 2010, John Wiley ans Sons, Inc.

W. R. Leo — **Techniques for Nuclear and Particle Physics Experiments: A How-To Approach**

2nd edition, 1994, Springer-Verlag

Review of Particle Physics (2014) – mainly section 32

K. A. Olive et al. (Particle Data Group), Chinese Physics C, 2014, 38(9):090001.

biennial update